COGNITIVE NEUROSCIENCE AND MATH EDUCATION: TEACHING WHAT KIDS DON'T LEARN BY THEMSELVES

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ABSTRACT

The aim of the present article is to review contributions from neuroscience to a better understanding of mathematical learning processes. A review of the theoretical frames and methodological assumptions from the developmental cognitive neuropsychology is firstly introduced. Then we analyzed the evidences from the neuroscientific investigation of typical and atypical development of mathematical abilities and the implications of those evidences in the context of mathematics education. Finally we conclude that the interactions of neuroscience and mathematics education may benefit all students, by thinking of the cognitive mechanism underlying the development of mathematical abilities.

Keywords: neuroscience, mathematical cognition, neuropsychology, mathematical education, Approximate Number System

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INTRODUCTION

Interest in collaboration between neuroscience and education is growing (Chinn & Ashcraft, 2007; Posner & Rothbart, 2007). Potentially, both areas benefit from the partnership. A neuroscientifically informed pedagogical approach might help improve teaching of both typically developing children as well as those with learning difficulties. Interdisciplinary work between educators and neuroscientists may result in a broader and more reliable diagnosis of the students' individual differences in terms of cognitive assets and deficits, contributing to develop evidence-based intervention strategies. From the neuroscientists' perspective the dialogue with education may constitute an opportunity to balance inventory, contrasting and assessing assumptions, methods and validity of empirical results. In the process of accommodation to the theoretical and practical contributions and needs of educators, neuroscientists main gain insights and opportunities of renewal for their own field.

The modernization of society demands a larger coverage of middle and higher education and better quality teaching (Newsom & Richerson, 2008). The educational level of the population has an impact on the form of human, social and mental resources (Barber, 2002; Cooper, Field, Goswami, Jenkins, & Sahakian, 2010; Fukuyama, 1996, 1999), as well as on quality of life (Felder-Puig, Baumgartner, Topf, Gadner & Formann, 2008).

Perhaps the most important aspect to integrate neuroscience and education is to think about the cognitive mechanisms underlying learning processes (Coch & Ansari, 2009). So the aim of the present article is to present contributions from neuroscience to a better understanding of mathematical learning processes. Firstly, we will start reviewing the theoretical frames and methodological assumptions typical of neuroscience. It is necessary to clear from the beginning, that the field of neuroscience is extremely heterogeneous, and that we are adopting a more specific perspective, that of developmental cognitive neuropsychology (Temple, 1997). Secondly, we will present evidences from the neurocientific investigation of typical and atypical development of mathematical abilities and analyze the potential applications of those evidences to basic arithmetics education. Finally psychosocial aspects of mathematical education will be briefly discussed.

DEVELOPMENTAL COGNITIVE NEUROPSYCHOLOGY

Clinical neuropsychological evidence suggests that the brain may be characterized by a modular architecture. That is, the cognitive system is built from a sample of evolved processing devices, which are both integrated and partially autonomous and functionally segregated. Inherent biological interindividual variation or pathogenic processes may compromise the host of modular systems both globally, as in mental retardation, or isolated, as in specific learning disabilities (Anderson, 2001). Developmental cognitive neuropsychology does not deny either the complex and integrative nature of the brain or the existence of an important environmental influence on the maturation of cognitive systems, but assumes that analysis of neuropsychological dissociations in performance of brain-injured or brain–dysfunctional individuals constitutes a valid approach to identify the semi-autonomous components that constitute the system (Shallice, 1988; Temple, 1997).

The idea is to analyze the differing patterns of lost and preserved psychological processes arising from distinct forms of brain damage or dysfunction in search for double dissociations (Shallice, 1988; Temple, 1997). Some individuals, for example, are identified as developmental dyslexics, because they present with specific deficits in the phonological recoding processes underlying reading learning, while their visuospatial, visuoconstructional, arithmetical and socio-cognitive inferential abilities may be intact (Galaburda, LoTurco, Ramus, Fitch & Rosen, 2006). In another group of individuals, phonological recoding abilities related to literacy acquisition may be spared, but important difficulties are experienced with visuospatial, visuoconstructional, arithmetical and socio-cognitive performance, being diagnosed with the so called nonverbal learning disability (NLD, Davis & Broitman, 2011; Rourke, 1989; Volden, 2004). The complementary profiles of spared and compromised neuropsychological processes observed in developmental dyslexia and NLD may be referred to as a double dissociation, being interpreted as evidence for a modular organization of the brain.

Obviously, the modular organization of the brain, as observed in adults, is the outcome of a long epigenetic process, in which, synaptic plasticity, environmental structure, and system dynamics all work as powerful influences. But, it is argued, the double dissociation between distinct profiles of "basic phonological processing" and "nonverbal" learning disabilities (Rourke, 1989) indicates that the initial system state may not be characterized as lacking structure, being more adequately described by differential learning dispositions regarding proclivity to acquire certain abilities and not others. The genetic nature of learning disabilities (Galaburda et al., 2006; Shalev et al., 2001) is consistent with the hypothesis that minimal differences in the initial conditions of a complex system may be amplified and lead to distinct outcomes.

Persistence of learning disorders across the lifespan demonstrates that neuroplastic processes know some important scope limitations (Anderson, Catroppa, Morse, Haritou & Rosenfeld, 2005). Disorders of synaptic neuroplastic mechanisms may build the core mechanism of developmental/learning neurogenetic disabilities, impairing learning and development in more or less specific domains (Johnston, 2003). From the specificity of the impairments, it is possible to suppose that some individuals may exhibit unique learning strategies or styles, which need to be considered in the educational process (Chinn & Ashcroft, 2007). For example, individuals with the NLD syndrome lack intuition and do not benefit so much from the social interactive context as their peers (Davis & Broitman, 2011).

Evolutionary psychology has an important theoretical argument for mental modularity. If the cognitive system is the outcome of an ancestral evolutionary process by natural and sexual selection, then it must be modularly organized, as it is difficult to conceive how a general solving device could be selected (Tooby & Cosmides, 1995). In the modular-evolutionary perspective, each cognitive process represents an adaptation to a specific problem found by our ancestors in the environment in which our species evolved.

Another source of evidence in favor of modularity comes from recent investigation on mathematical models of topology, which are known as small world models and graph theory (Bassett & Bullmore, 2006). This line of investigation shows that a certain degree of modularity in complex systems is necessary to allow neural specialization. Moreover, complex systems with a modular structure do not react to evolutionary influences to the same extent and are not changed at the same speed. As a consequence, efficient cognitive subsystems can preserve their structure while less efficient subsystems may be changed faster depending on environmental pressure till they reach a higher degree of efficiency. Meunier, Achard, Morcom and Bullmore (2009a), for instance, have shown with graph models that aging has a much larger impact on the structure of fronto-parietal connections than on the functional connectivity of visual occipital structures, which remain largely unchanged across adult life. Graph models are also useful to distinguish between nodes presenting high intra-modular connectivity from other regions presenting high inter-modular connectivity (Meunier et al., 2009b). While the first type of node is particularly important for integrating information within a given module and to increase the efficiency of a specific cognitive function such as vision or reading, the last type of node integrates high-level information from different cognitive modules. As stated by Meunier et al. (2009b), a modular structure equips cognitive systems with both the stability and flexibility typically associated with well-adapted complex systems.

Modularity also helps to understand why not every animal species learns equally easy or well any single considered behavior (Garcia, Kimeldorf & Koelling, 1955); the system's initial state must then be accounted as consisting of several learning dispositions, which increasingly differentiate themselves during ontogenesis. Preparedness or initial dispositions may be characterized as a set of innate intuitions, which allow adaptation to distinct characteristic behavior of most species, as well as human social and cultural behavior (Geary, 2007; Spelke & Kinzler, 2007). The ensemble of primitive intuitions varies, but different formulations gravitate around a set of abilities related to psychosocial knowledge, of self and others, knowledge of living nature, as well as knowledge of physical phenomena such as space, motion, causation, and a rudimentary representation of continuous and discrete quantities (Geary, 2007; Spelke & Kinzler, 2007).

In the domain of arithmetics, a distinction was proposed between biologically primary and secondary abilities (Geary, 2007). Primary abilities correspond to the primitive intuitions of number and arithmetical principles, which are spontaneously acquired by children in their interactions with the physical and social world, and which, ordinarily do not require more elaborate pedagogical interventions to be acquired. They are characterized as representing *start-up tools* with which arithmetic knowledge is built (Piazza, 2010). Secondary abilities, on the other hand, are exemplified by cultural developments, such as the multiplication tables, the Arabic notation and algorithmic procedures. Cultural tools require an explicit educational intervention to be learned, besides training and effort to be mastered. The "natural" or spontaneous acquisition of cultural tools, by mere exposition, is considerably rarer.

In the second part of this article we will review five different domains that neuroscientific research can contribute to better understand the typical and atypical development of mathematical learning processes: 1) approximate number system (ANS) and the development of the concept of number, 2) fact retrieval, 3) procedural knowledge, 4) conceptual knowledge and 5) emotional reaction to mathematics. The ANS is an evolutionary selected capacity to perceive and represent nonsymbolic numerical quantities (Feigenson, Dehaene & Spelke, 2004). Fact retrieval refers to simple arithmetic problems learned by heart such as multiplication tables (Dehaene et al., 2003). Procedural knowledge is the ensemble of heuristics and procedures employed to dismember complex arithmetic problems into its components with the aim of solving them (know-how). Procedures are ensembles of rules describing what to do when facing specific kinds of arithmetic problems such as simple binomials $(a + b)^2$. The set of transformations necessary for solving a problem are the core of procedural knowledge. In contrast, conceptual knowledge is the set of general logical principles ruling mathematics. Knowing that 22 + 75 = 97 helps solving the problem 97 - 22 is a clear example of conceptual knowledge. Finally, the emotional reaction to mathematics is one important predictor of successful mathematics learning. Math anxiety is a condition that may prejudice arithmetic learning (Krinzinger et al., 2009).

ANS AND THE DEVELOPMENT OF THE CONCEPT OF NUMBER

One of the most important breakthroughs was the discovery of the existence of a *number sense*, that is, a fundamental kind of intuition shared with other animals and operating since infancy to represent and process numerical magnitudes. (Dehaene, 1999). These foundational abilities are developed by human children in their natural environment and typically require minimal pedagogical intervention.

Other abilities, related to verbal and Arabic symbolic numerals require extensive teaching, years of practice and considerable cognitive effort to achieve mastery.

The well-known experiments by Piaget in the first half of the XXth Century with quantity conservation suggested that children only acquired a concept of number as cardinality after 7 or 8 years of age (Piaget & Szeminska, 1975). A series of investigations in the last decades, however, using nonverbal experimental paradigms of habituation or anticipation, suggests that even infants are endowed with a rudimentary, nonverbal and approximate notion of numerosity. Results have shown that infants in their first weeks or months of life are able to discriminate the cardinality of small sets up to four elements (Starkey & Cooper, 1980).

Wynn (1992) had demonstrated that infants under 5 months of age are able to conduct, in a nonsymbolic form, rudimentary operations of addition and subtraction. Subsequently, Xu and Spelke (2000) showed that infants could also approximately but reliably discriminate the cardinality of larger sets and that their response pattern respected the scalar variability property described by the Weber's law. Research showing that infants are able to discriminate small numerosities and also of anticipating results of simple arithmetic operations suggest that these abilities may have an innate basis.

Discovery of infants' numerical and arithmetic abilities elicited a vivid debate in the developmental literature, concerning the nature of the involved representations, e. g., if they are specifically numerical and discrete, or if they are continuous and shared with other magnitude representations systems (see Rousselle, 2005, for a review). One of the main critiques pertains to the lack of experimental control of all possible perceptual dimensions which can confound performance (Mix, Huttenlocher & Levine, 2002).

In a carefully designed study, however, Feigenson, Carey and Spelke (2002) contradicted this interpretation showing that infants could better discriminate numerosity than surface. Another experiment, conducted by Wynn, Bloom e Chiang (2002) contributed to firmly establish the notion that 5-month-old babies can discriminate the numerosity of sets. Stimuli were sets of moving points. In the habituation phase, infants saw two sets of three points each. In the experimental phase, two kinds of stimuli were shown, both with the same total amount of eight points: two sets of four elements or four sets of two elements. Infants kept their gaze fixated on the condition with four sets for more time, demonstrating that they were attending to the numerosity of sets.

Nonetheless it remains possible that early magnitude representations are not specifically numeric or discrete in nature, being continuous and shared with other magnitude representation systems in the parietal lobes, such as time and space (Walsh, 2003). According to this perspective, the concept of cardinality and intuitions on the principles underlying arithmetic operations are acquired epigenetically. One possible mediator between the ANS and the full-fledged concept of cardinality could be fostered by counting mechanisms recruiting the verbal labels, finger-pointing and finger-counting procedures under visual attentional control (Lecointre, Lépine & Camos, 2005).

Consistent experiments conducted with and humans adults demonstrated that discrete numerical processing also obeys traditional psychophysical laws, such as the ones described by Weber and Fechner in the XIXth Century. Moyer and Landauer (1967) observed, for example, that the responses to comparison of the magnitudes of two Arabic numbers are slower and more prone to error when the numerical distance between the compared numbers is smaller, than when this difference is larger (distance effect; see Sekuler & Mierkiewicz, 1977, for a description of the distance effect in children). This proportionality or scalar variability between magnitude differences and their discriminability in several domains was discovered by Weber to be a constant. Number processing in animals and humans accords also to another psychophysical regularity described by Fechner (Dehaene, 2003), that is, is progressively slower and error prone as the magnitude of the stimuli increases, and the function which best fit to the data is logarithmic (size effect).

Evidence for the ANS led Dehaene (1992) to formulate the triple code model, the most widespread contemporary used model of number representation, processing, and calculation (see Figure 1). According to the triple code model, number processing and arithmetic operations may be conducted on three systems of mental representations: the ANS and two symbolic systems, which allow for precise numerical representations of magnitude. Numerical symbolic systems consist of the verbal numerals (in phonological or orthographic form) and visual Arabic digits.

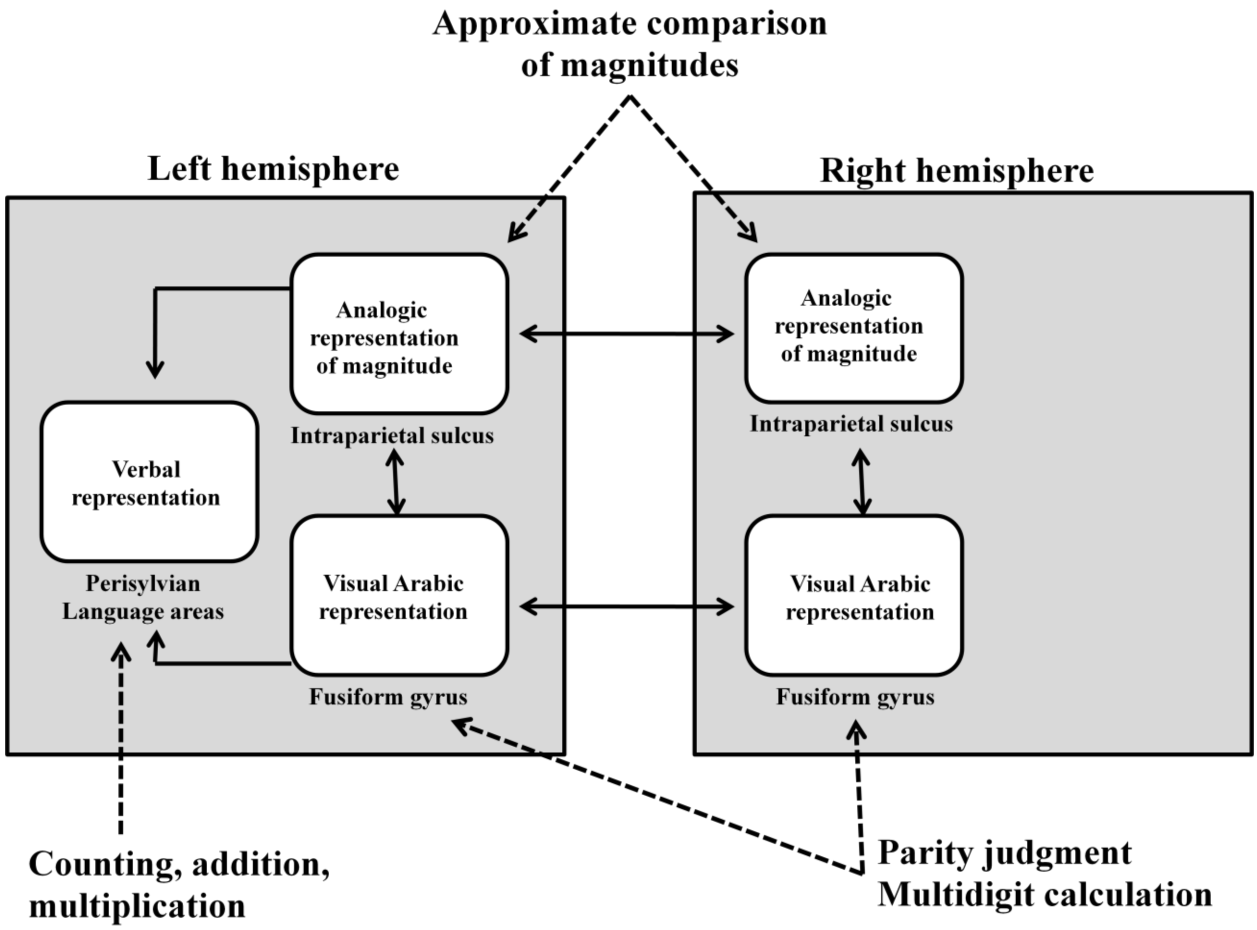

Figure 1 – Dehaene's triple code model.

A series of neuropsychological studies lead to the formulation of the neural underpinnings of the triple code model (Dehaene & Cohen, 1995), which were subsequently confirmed by neuroimaging research (Dehaene, Piazza, Pinel & Cohen, 2003). Processing of verbal numerals is implemented by perisylvian regions of the left hemisphere, most notably the region around the angular gyrus. Processing of Arabic numerals is postulated to depend bilaterally on the region of the fusiform gyrus, the occipito-temporal ventrolateral border. Bilaterally situated neuronal networks around the horizontal portion of the intraparietal sulcus may constitute the neuronal substrate the ANS (Walsh, 2003). Strategic aspects of number processing depend on the dorsomedial and dorsolateral regions of the prefrontal cortex and related circuits. Proceduralization of arithmetic facts takes place via interactions between circuits comprising the before mentioned regions and subcortical basal ganglia structures, resulting in a specific domain of semantic memory, represented in widely distributed form in several cortical areas, but having the angular gyrus as a kind of hub or portal of access (Zamarian, Ischebeck, & Delazer, 2009).

Taken together these studies demonstrate that humans have an innate ANS which serves as a foundational capacity to the development of the concept of number.

FACT RETRIEVAL AND LEARNING BY HEART

Sometimes the contribution of neuroscience comes in a counterintuitive way. One very clear example is learning by heart. The more progressive the educational attitude, the more learning by heart is despised. Learning the general concept is seen as much more valuable than mechanically repeating contents (see Bryant & Nunes, 2011, for a contemporary reformulation of the constructivistic approach to mathematics). Traditionally, constructivistic approaches in education favor learning over fluency acquisition, which sometimes leads to curricular distortions, insofar as learning by heart is regarded as old fashioned and inefficient, being relegated to a secondary position in curriculum (Fuson, 2009). Students struggling with difficulties learning arithmetics are the ones most adversely affected by such biases. Interestingly, there seems to be important exceptions to this general rule with regard to mathematics: multiplication tables.

The cognitive mechanisms responsible for storing and correctly retrieving multiplication facts are not sensitive to conceptual learning or learning transfer. To the contrary, these mechanisms are specialized in storing and retrieving exactly the material learned. The efficiency with which these mechanisms work depends on the degree of automaticity to which memory traces can be retrieved. Moreover, retrieval efficiency depends mainly on the material being learned and show therefore no effect of generalization. For this reason, learning very well the multiplication of 3 will not help learning the table of 7 or 8.

The brain regions related to learning and retrieval of multiplication facts differ from the regions typically involved in magnitude estimation and calculation. Neuropsychological evidence so far is largely compatible with the proposed structural-functional correlations assumed by the triple code model, such as double-dissociations in performance observed in several case studies (Dehaene & Cohen, 1997; Delazer & Benke, 1997; Delazer, Karner, Zamarian, Donnemiller & Benke, 2006; Lerner, Dehaene, Spelke & Cohen, 2003; Varley, Klessinger, Romanowski, & Siegal, 2005). Several patients presented with specific impairments in the processing of verbal numerals, preserving performance in tasks related to nonsymbolic magnitudes, such as magnitude comparison of visually presented sets of points. And, vice-versa, other patients preserved performance on verbally mediated tasks, such as knowledge of multiplication tables, in the presence of impairments in their ability to estimate the cardinality of object sets. In general, verbal dysfunctions disproportionately disrupt addition and multiplication operations, while disruption of the ANS interferes with subtraction and approximate calculation. A broader review of neurofunctional and clinical aspects related to calculation disorders may be found in Willmes, 2008. Broadly speaking, localized lesions of the left hemisphere in the region of the angular gyrus cause acalculia by interfering with verbal mechanisms, while bilateral damage to parietal structures, generally in the context of developmental or degenerative diseases are a cause of malfunction of the ANS.

Interestingly, learning by heart is important mainly for multiplication tables but not for any other basic arithmetic operation. Therefore, it is very important to practice the retrieval of multiplication problems in a mechanical way.

In summary, learning by heart is central for efficient learning of multiplication facts. Children who do not recite and repeat multiplication problems enough in the first years of schooling will be less efficient solving these sorts of problems in the future. This is indicative of the diversity of approaches and methods necessary for learning mathematics, which go far beyond the ability of transfer from one context to the other (Soistak, Pinheiro, Galera, 2008).

PROCEDURAL AND CONCEPTUAL KNOWLEDGE: KNOWING THE HOWS AND KNOWING THE WHYS

The main goal of basic mathematical education is related to the acquisition of "understanding" or adaptative expertise required to solve arithmetic problems posed by daily life (Baroody, 2003). Automatized, routine arithmetical skills related to knowledge of arithmetic facts and algorithms are generally regarded as secondary, or instrumental. For example, resolution of multidigit calculation or word problems requires automated knowledge of both math facts and algorithms. Neuropsychological evidence suggests, however, that the knowledge of arithmetical skills and concepts are at least partially segregable and, although interactive, do not causally related to each other. The message being that procedural knowledge does not follow from mastery of arithmetic principles.

Under conceptual knowledge, we understand both biologically primary and secondary forms of abilities. One of the most basic forms of conceptual knowledge is represented by the principles of counting. Interacting with the physical and social environment, most preschool children develop an intuitive grasp of the verbal counting procedures and their relations to ordinality and cardinality, thus enabling to begin using counting as a strategy to solve simple arithmetic operations (Gallistel & Gelman, 1992). Practicing simple computations through counting, some children can effectively induce the principles underlying arithmetic operations, such as additivity and associativity that characterize addition. But most children require formal schooling and instruction in order to grasp and master arithmetical principles underlying the four operations (see review in Geary, 2006).

After an initial conceptual grasp that counting may be used to compute simple arithmetic problems, children progressively employ several counting strategies to solve addition problems, such as counting all items in a set, counting from the larger cardinal value (max strategy) etc. Different problem-solving strategies are successively employed in a series of overlapping waves, until children associatively acquire arithmetic facts and begin to retrieve solutions from long-term memory (Siegler & Shrager, 1984). When they begin using counting to solve addition problems, children commit several procedural errors, such as missing to count one element in the set, counting one element twice, etc. That is, they have the conceptual knowledge, but fail in its application (Geary, 2006). Perfectioning in strategy application requires practice and constitutes a kind of procedural knowledge.

Procedural knowledge may be considered a form of routine expertise (Baroody, 2003), in that it is automatic, mandatory and not necessarily consciously aware.

Practice with simple addition problems leads to progressive development of two arithmetic skills: arithmetical facts and computation procedures. Available evidence indicates that both facts and procedures require extensive practice and develop by means of associative learning involving synaptic plasticity in critical brain regions. Computations are initially carried on under controlled or executive forms of processing. Working memory resources represent the limiting factor in the early learning of arithmetic computations. Functional neuroimaging studies show that, initially prefrontal areas are the predominant loci of activity. As the individual progressively automatizes arithmetic operations, activity loci move posteriorly and subcortically (Rivera, Reiss, Eckert & Menon, 2005; Zamarian, Ischebeck & Delazer, 2009). The critical loci of assumed synaptic plasticity differ between math facts and operation procedures. Available data indicate that the left angular gyrus is progressively activated as individuals acquire proficiency in new math facts (Zamarian et al., 2009).

Less information is available on the structural-functional correlations of arithmetic operations, but there is reason to suppose that some regions of the basal ganglia, such as the dorsal striatum, are implicated. Adaptive alterations in firing patterns of tonic active neurons in the striatum have been for many years implicated in the operant learning of habitual behavioral sequences in animals (Graybiel, Aosaki, Flaherty & Kimura, 1994; Pennertz et al., 2009). Neurological diseases such as Parkinson's and Huntington's have been long characterized by both general (Knowlton, Mangels, & Squire, 1996) and specific arithmetical (Teichmann et al., 2005; Zamarian et al., 2006) procedural deficits. Neuropsychological evidence, thus, suggests that the dorsal striatum is involved in the creation of habits, routines, or sequentially organized repetitively reinforced operations involved both in syntactic linguistic and arithmetical algorithmic processing.

Written multidigit calculation is an arithmetic skill that is suited to illustrate the interplay of conceptual and procedural knowledge in learning arithmetic. First of all, multidigit calculation requires grasping the concept of base-10 place value, but it is at the same time enormously facilitated by the algorithms made possible by the Arabic notation. Efficient use of the Arabic algorithms requires extensive practice and several abilities, related to math facts, such as transcoding between notations, alignment of digits, borrowing and carrying operations, etc. All steps involved in the execution of Arabic algorithms are initially controlled by working memory and become progressively automatized (Rivera et al., 2005). Knowledge of place value is simply not enough, as children continue to commit procedural errors until they proceduralize such knowledge (Geary, 2006).

Neuropsychological evidence obtained with acquired and developmental calculation disorders both in adults (Granà, Hofer & Semenza, 2006; Hittmair-Delazer, Semenza & Denes, 1994; Semenza, Miceli & Girelli, 1997) and children (Murphy & Mazzocco, 2008; Temple, 1991) demonstrates that procedural and conceptual arithmetical knowledge are dissociable components, being modularly organized. Hittmair-Delazer and coworkers (1994) described the case of patient with deficient facts and preserved conceptual knowledge. The patient was able to use back-up strategies based on conceptual knowledge to effectively solve arithmetic operations. Rehabilitative efforts by the authors showed that conceptual knowledge was of little help in improving arithmetical fact knowledge, which required extensive training to be partially reacquired in very limited and specific way. Cognitive-neuropsychological case studies show, otherwise, that procedural knowledge may be selectively impaired with preservation of conceptual knowledge. Patients were described with acquired acalculia, in whom knowledge of the algorithmic multidigit calculation procedures was loss (Girelli & Delazer, 1996), who had difficulty monitoring the implementation of algorithms (Semenza et al., 1997), and who predominantly committed errors of spatial nature, such as beginning all operations from the left to right, with impairment of all operations except division (Granà, Hofer & Semenza, 2006).

One could, however, argue that cognitive-neuropsychological dissociations observed in acquired disorders demonstrate that conceptual knowledge may be preserved in face of severe disorders of multidigit calculation procedures. This hypothesis is contradicted by several observations. The most relevant evidence comes from the work by Murphy and Mazzocco (2008). These authors have shown that math difficulties in girls with fragile-X syndrome are characterized by a pattern of relatively preserved math facts and simple algorithms in face of severe conceptual difficulties. In other words, girls with fragile-X syndrome can learn the facts and execute some simple algorithms, but experience considerable difficulty understanding what they are doing.

DEVELOPMENTAL DYSCALCULIA

As the distribution of every psychobiological trait or characteristic is widely dispersed in the population, there is a potential for dysfunction or environmental maladaptation in the extremes (Wakefield, 2007). Developmental dyscalculia (DD) is a specific learning disability in which an individual of normal intelligence experiences disproportionate difficulties with math achievement, which cannot be ascribed to emotional factors or inadequate schooling (Ansari, 2008; Dehaene & Cohen, 2007; von Aster & Shalev, 2007). Dyscalculia is a persistent and striking disorder (Shalev, Manor & Gross-Tsur, 2005), which potentially compromises the individual’s well-being and mental health (Auerbach, Gross-Tsur, Manor & Shalev, 2008) and also its proper integration in the society (Geary, 2000). In increasingly informatized and cognitively sophisticated societies, even health self-care is affected by the level of math literacy (Estrada, Martin-Hryniewicz, Peek, Collins & Byrd, 2004). The DD prevalence is estimated as being around 3-6% of the school age population (Shalev, Auerbach, Manor & Gross-Tsur, 2000).

DD is a clinically and cognitively heterogeneous condition. There is evidence of substantial comorbity with other learning and behavioral disorders such as dyslexia and attention deficit/hyperactivity disorder (Rubinstein & Henik, 2009). A host of cognitive factors has been implicated in the genesis of mathematical learning disorders, such as phonological abilities (Simmons & Singleton, 2008), working memory (Geary, 2011) and ANS (Mazzocco, Feigenson & Halberda, 2011; Piazza et al., 2010). An important clue to the genetic origin of mathematical learning difficulties stem from the fact that several neurogenetic syndromes, such as Turner’s (Bruandet, Molko, Cohen & Dehaene, 2004) or velocardiofacial syndrome (de Smedt, Swillen, Devriendt, Fryns, Verschaffel & Ghesquière, 2007), present severe and disproportionate difficulties learning arithmetics as an important phenotypic trait.

Research on the neurocognitive bases of mathematical learning difficulties is ongoing, but some generalizations seem possible at the moment. Although its precise nature is still not entirely clear, evidence indicates that cognitive deficits are relatively specific. That is, difficulties learning mathematics cannot be entirely ascribed to general cognitive factors, such as intelligence, working memory, or verbal ability (Butterworth & Reigosa, 2007). Demonstration that children with dyscalculia have difficulties in tasks such as nonsymbolic magnitude comparison suggests that, at least in some phases of development, number sense may be important to acquire mathematical concepts and procedures (Mazzocco, Feigenson & Halberda, 2011; Piazza et al., 2010).

Genetic factors also account for a substantial proportion of variance in math achievement, both in the general population as in individuals with more specific genetic disorders (Willcutt et al., 2010). As socioeconomic factors also play an important role in math achievement (Jordan & Levine, 2009), children of lower strata incur in the risk of being double handicapped. If any child living in poverty is genetically prone to mathematical learning difficulties, the individual's liability may be enhanced when his/her difficulties are not properly recognized and treated.

Research on mathematical cognition and associated learning difficulties suggests that arithmetic is a complex domain, characterized by multiple, interacting but partially independent subcomponents. Each domain is implemented by specific neural underpinnings subject to both genetic and environmental sources of variation. The etiology of arithmetical performance is multifactorial (Willcutt et al. 2010), there being no grounds to assume qualitative mechanisms differences between lower and higher performance (Mazzocco, 2007). Labelling of individuals in the lower performance end of the continuous as carriers of a disorder is arbitrary and justifiable only because low performance is persistent (Shalev et al., 2005), consistently associated to unfavorable outcomes, requiring proper intervention. Research has shown, for instance, that, DD is a predictor of behavioral and emotional disorders (Auerbach et al., 2008), and unemployment and lower wages (Bynner & Parsons, 2006; Rose & Bett, 2001; see Haase, Moura, Pinheiro-Chagas & Wood, 2011, for a review).

As the distinction between normal and abnormal arithmetical performance is statistically arbitrary, broadly varying from one study to the other, Mazzocco (2007) proposed a terminological convention, which is being increasingly adopted, and which enormously facilitate studies comparability. According to Mazzocco, children whose arithmetical performance was identified as being under the 25th (or event the 35th) percentile can be characterized as having mathematical difficulties (MD). Research suggests that low performance in children identified as MD is not temporally stable, being also subject to a diverse range of etiologies, including state variations, and social and emotional factors. Using a stricter criterion of performance under the 5th percentile it is possible to identify a smaller group of children, whose performance difficulties are chronic in nature and whose etiologies are probabilistically related to inherent factors of probable neurogenetic origins. Following Mazzocco (2007), this latter group of children can be labelled as DD or mathematical learning disability (MLD). Research on both sides of the arithmetical performance arbitrary divide is complementary, contributing to identify the neurocognitive foundations to arithmetic, being of enormous consequence to pedagogy, as it will be discussed in the next section.

NUMBER SENSE TRAINING

The relations between the number sense and mathematics achievement, as well as its involvement in DD occurrence, have been well established over the last years (Butterworth, Varma & Laurillard, 2011; Landerl, Bevan & Butterworth, 2004; Landerl, Fussenegger, Moll & Willburger, 2009; Halberda, Mazzocco & Feigenson, 2008; Mazzocco, Feigenson & Halberda, 2011; Mussolin, Mejias & Noel, 2010; Piazza et al., 2010). It has been argued that an appropriate training of basic number sense abilities would induce an improvement on basic and also on more complex mathematical tasks. Main inspiration comes from dyslexia research, which showed that specific training on the core process of phonological awareness improves general performance on more high-level abilities (for an example, see: Torgesen, et al., 2001). In despite of that, research on number sense based on intervention is still on its earliest times.

One of the first contributions regarding number sense training effects was given by Dehaene and his collaborators with the software *Number Race*, a computer game developed to work as a training tool for children with low arithmetic performance (Wilson, Dehaene, et al., 2006; Wilson, Revkin, Cohen, Cohen, & Dehaene, 2006). In a playful environment children compete against the computer on a variety of early arithmetic aspects, like comparing numbers and set of dots (choose the larger), counting, linking symbols to concrete quantities, and simple calculation. The game was open-source made, so it is free for use and transform (www.unicog.org). Their first results indicated a significant improvement in performance on the more basic numerical cognition tasks, like number estimation and comparison, and also on subtraction, after the training period (Wilson et al., 2006b).

Other results come from studies conducted by Siegler and coworkers (Ramani & Siegler, 2008; Siegler & Ramani, 2008, 2009), who showed in more detail how the training works on preschoolers. Their program is base on the strengthening of number-space relationship representations by means of classical numerical board games. According to their results, numerical board games practice is capable to improve performance on numerical magnitude measures and, therefore, reducing individual differences between preschoolers on those abilities. Children with lower performance on numerical knowledge tasks exhibited, after the training epoch, performance levels similar to that of children with previous high performance on a variety of number tasks, especially magnitude estimation, digit naming, addition and magnitude comparison. The best results were found when comparing children from discrepant socioeconomic neighborhoods. In this sense, core number knowledge training is capable to fill the gap existing on the numeric-related stimulation provided by their home and social environments, strengthening the link between symbolic and nonsymbolic representations of number. Additionally, one of their most outstanding discovers was the training effect on subsequent arithmetic learning. Children who first had practiced on linear board game exhibited better results on subsequent arithmetic problems learning, indicating that early number magnitude training has a cumulative effect on later mathematical comprehension (Siegler & Ramani, 2009.

In a recent study Kucian and coworkers (2011) evaluated the efficacy of a computer-based training program on DD rehabilitation by means of neuropsychological testing and functional magnetic resonance imaging (fMRI). Computer training was very similar to a regular video-game (with challenging goals and a motivating story) where the player should associate numerical stimuli (Arabic numbers, dot sets or simple calculation) to the correct location on a number line. After training, both magnitude representations and neurofunctional changes were investigated. Significant performance improvements after test-retest were found for number line estimates and arithmetic achievement for both groups, but more relevant for the DD one. Regarding neurofunctional effects of training program, results point to a reduction in the activity levels of magnitude processing related areas (bilateral intraparietal regions), suggesting an automation of the cognitive processes related to numerical competence.

Another approach, proposed by Gilmore, McCarthy and Spelke (2007), aimed at developing word problem solving abilities. These authors worked with preschool children, prior to any formal arithmetic instruction. They presented children with word arithmetic problems on a computer program, which required simple addition or subtraction procedures to manipulate sets of visually displayed objects such as candies. Children were encouraged to estimate the answer without counting. Experimental conditions manipulated magnitude distances between the sets to be operated on. Quantitative differences between operand were initially large, being progressively reduced. With training, children progressively improved their ability to provide increasingly precise answers and developed an intuitive grasp of the problem-solving strategy. The study by Gilmore and coworkers suggests that nonsymbolic approximate estimation operations may play a role in the development of arithmetic problem solving if the magnitudes involved in the problems are large enough to be easily discriminated.

Together, these studies constitute the first main progress regarding the remediation of core numerical abilities. Unfortunately, results still should be interpreted carefully due to reduced sample sizes and limited methodological designs. Nonetheless, evidence points to the efficacy and reliability of activities that deal with basic mathematical concepts like magnitude estimation, counting and number reciting, and also regular board games, on both school and home contexts. It is clear that core numerical abilities stimulation, on the very first school years, is capable to remediate and improve current numerical performance, and also has a long term effect on later mathematical learning.

EMOTIONAL AND PSYCHOSOCIAL ASPECTS OF THE MATHEMATICAL EDUCATION

Mathematics anxiety

Up to this point problems with numbers and arithmetic have been described from a cognitive point of view. Nonetheless, mathematics difficulties impact also on emotions, especially on negative ones. Who has never feared complex mathematics problems in a test, who did never experience helplessness when the time is running faster than one can solve enough problems to pass a mathematics test? Most of us has experienced that at least once or twice in school or later on in college. Moreover, how many of us can say, that they liked not only the topics treated in mathematics classes but also the mathematics teacher? Only a few, for sure. Most of us used to experience more anxiety before a mathematics test than before tests in any other topic. Moreover, the preparation for these tests used to consume more time and effort and of course more self-control and tolerance against frustration than other topics.

Since at least one decade, modern neuroscience and cognitive psychology have been investigating the causes of these negative feelings towards mathematics. The fear of mathematics has also been called "math anxiety". Math anxiety describes the negative stress responses associated with learning and being tested in the topic mathematics. Math anxiety is more pronounced in girls than in boys (Dowker, 2005). Interestingly, math anxiety is not strongly correlated with performance in mathematics. There are almost as many students with a very high mathematics performance and high math anxiety as students with low performance. As other forms of anxiety, math anxiety is associated with specific psychophysiologic stress responses which include an increase in autonomous responses and the liberation of stress hormone cortisol in blood-flow, which increases heart- and breath-rates. Stress responses have an interesting connection with performance: a certain amount of stress influences performance positively. In the case of mathematics, a little fear of having committed a mistake here or there can increase concentration and accuracy and thereby increase performance. However, too much stress may lead to panic reactions and to the sensation of having a blank. Contrary to the positive effect of a moderate amount of stress on performance, too much stress can have a dramatically negative impact on the outcome and lead to much worse marks in mathematics courses than that corresponding to the actual cognitive abilities of children.

Stress responses are learned, very resistant to changes, and can be very specific. This means that a student with high math anxiety may not necessarily experience any anxiety in other school topics. Therefore, it is very important for the education in mathematics to avoid unnecessary stress. To correctly allot the amount stress necessary to increase the motivation of a whole class up to the point of highest performance while not exceeding this point in the case of students with math anxiety requires specific teaching abilities and accurate diagnostics of each single student.

One of the most important behavioral responses to stress with an impact on teaching mathematics is avoidance. Stress is one of the most powerful negative motives for learning. The negative emotional responses associated with learning in determined topics determine whether children experiencing high levels of math stress are more prone to avoid or escape situations generating math anxiety, i.e. mathematics classes and mathematics tests. Children with high math anxiety experience more stress learning mathematics than children with lower levels of math anxiety. Math anxiety is not something that vanishes after finishing school. College students may present considerable stress before mathematics tests (Pletzer et al., 2010).

GENERAL DISCUSSION

The model presented in Figure 2 suggests that primary arithmetic abilities (analogic representation of numerosity, counting, arithmetic principles, etc.) interact with the environment to elicit a sequential process of acquisition of arithmetic abilities under the influence of formal schooling, in such a way as to allow the development of secondary abilities (e.g. Arabic notation, positional base-10 value, algorithmic procedures etc.). After its elicitation by epigenetic interactions, the model is deliberately depicted as linear and sequential. Linearity and sequentiality are properties, which correspond to the fact that arithmetic acquisitions are cumulative and hierarchically organized. Subsequent procedural and conceptual developments in arithmetic require a solid basis in previous acquisitions. And the nature of previous acquisitions is peculiar. Arithmetic is a subject matter in which learning depends on knowledge proceduralization (Lieberman, 2000; Zamarian et al., 2009). Learning of arithmetic is only possible by means of a strenuous process of automatization of conceptual and procedural knowledge by means of an extensive training program. Mathematical abilities are only consolidated when the individual develops an intuitive knowledge, that is, mathematics must become second nature, otherwise is not learned. As an example, discovery of arithmetic principles seems to be based on the interaction of verbally mediated counting strategies and more primitive ANS (Lecointre et al., 2005). Mastery of the Arabic notation, by its turn, requires the child to be proficient in verbal abilities, composition principle and spatial representational abilities, which allow the comprehension of decimal positional value (Lochy & Censabella, 2005).

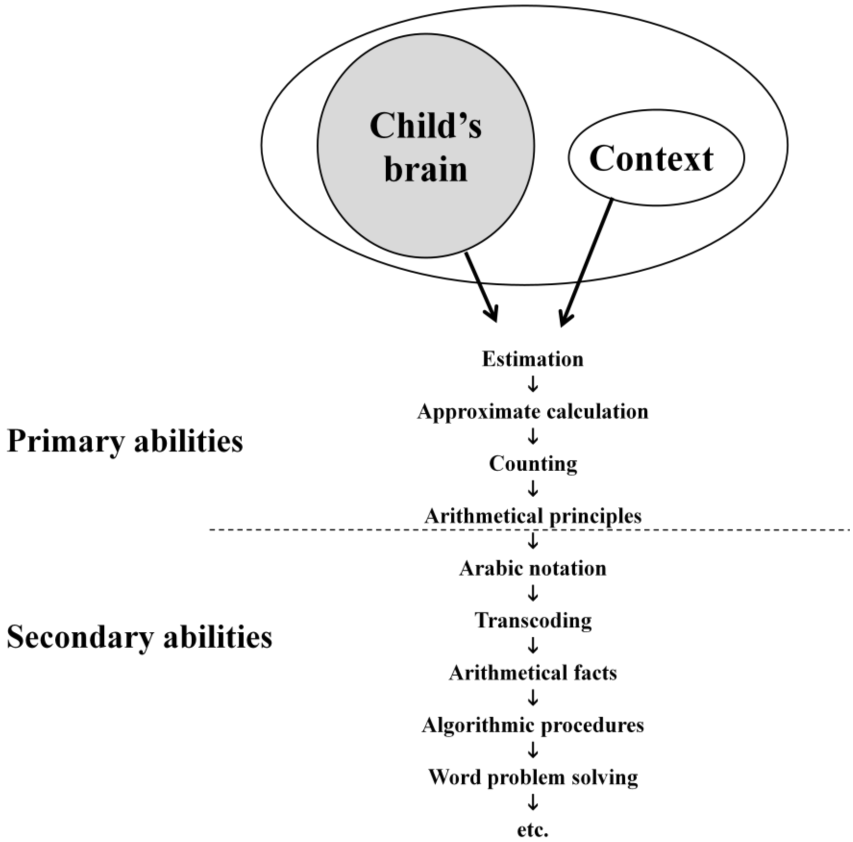

Figure 2 – Primary and secondary arithmetic abilities.

The considerations we have made conduct us to the argument that mathematical education may benefit from neuropsychological and neurocognitive knowledge. Neuroscientific evidence reviewed suggest that acquisition of arithmetic abilities may be characterized by 1) a modular albeit plastic and environmental interactive organization of the pertinent representations, 2) a complex interactive process between genetic and environmental influences in distinct developmental phases and across multiple levels of control, 3) interaction between symbolic and nonsymbolic, both automatically and deliberately activated representations, 4) the progressively cumulative nature of acquisitions, and, finally 5) the need to develop automatization or intuition of the underlying principles and procedures in one level before advancing to the next.

Two important implications for math education may be inferred in case of neuropsychological and neurocognitive data is considered relevant. Math education may benefit from a reliable neuropsychological diagnosis of the learner's developmental level and cognitive style or, eventually, assets and deficits profile.

Available evidence suggests that mathematical achievement may be explained by socioeconomic factors, but individual differences also explain a substantial and unique portion of variance (Jordan & Levine, 2009; Willcutt et al., 2010). On the other side, neuroscientific knowledge also corroborates recommendations that the math curriculum should be modular and sequentially organized. Students should progress from one module to the other only after acquiring intuitive mastery over one specific domain.

If neuroscientific considerations are important for typical learners their relevance is even greater in the case of children experiencing difficulties learning mathematics. Interest in neurocognitive informed intervention is growing, but still limited (Dowker, 2004). An important research focus concerns screening and early diagnosis. Available evidence shows that, besides poverty, individual differences in number sense at preschool are predictive or risk of developing difficulties learning math (Jordan, Kaplan, Locuniak & Ramineni, 2007). A few intervention programs for children with difficulties learning math have been developed based on the concepts of number sense, modularity, and sequential hierarchical organization of the curriculum (Kaufmann, Handl, Thöny, 2003; Wilson, Revkin, Cohen, Cohen & Dehaene, 2006b; see also references in Butterworth et al., 2011). Results are preliminary, but encouraging.

The presence of a learning difficulty may trigger a significant impact in the quality of life of the children and their family. Children with learning disabilities may have an entire life course marked by the presence of an excluding and stigmatizing label of "school failure". Reading and writing difficulties are well known by educators and the families in general, which allow an early diagnosis and treatment. On the other hand, mathematical difficulties are often underestimated by parents and educators and mistaken as a lack of intelligence, laziness or lack of motivation to study. Children with mathematics learning difficulties often receive pejorative labels and are excluded in the school environment by teachers and their peers, which can bring serious consequences to the academic development and individual's psychosocial adaptation.

Considering these aspects, it is necessary for educators to investigate the possible factors related to the learning disabilities of their students, instead of attribute all difficulties to general features, such as lack of stimulation in the family. Especially when a child has a particular difficulty in mathematics, and not in other disciplines, this difficulty can not be attributed solely to the cultural context of that individual, since the lack of a rich environment might explain an impairment in the learning in general, but not a specific difficulty in a particular discipline. It is worthwhile to think about an educational practice based on scientific evidences, analyzing the prevalences of learning difficulties. Evidences reveal that about 3-6% of the population has DD (Shalev et al., 2000) and about 15% of children present a milder math learning difficulty (Mazzocco, 2007). Thus, for each group of 100 students, a teacher is likely to have about five pupils with DD and about 15 children with mild math difficulties, which can not be attributed to contextual factors, but to individual, cognitive processes underlying the learning of mathematics.

A theoretical proposal very influential in the field of mathematics education is the sociology of education, especially the Bourdieu's theory, which argues that the degree of success achieved by students throughout their school careers could not be explained by its personal features, such as biological or psychological characteristics, but are related to their social origin, which would put them in a position more or less favorable to the school demands (Nogueira & Nogueira, 2002).

One area that supports the importance of context for learning mathematics is ethnomathematics, which has gained strength in mathematics education. The Ethnomathematics seeks to value the mathematics content of different social groups and concepts constructed by informal student in your life outside school. This area emphasizes the development of critical technology against the scientific context in which the student is involved, providing students with tools that help them both in a critical situation analysis and the search for alternatives to solve the situation (Borba, 1990; Powell & Frankenstein, 1997).

However, there are several evidences indicating the existence of individual differences in learning (as mentioned above), including children that present learning difficulties.

The role of traditional learning methods such as memorization has recently been taken up by math educators who argue that the poor arithmetical performance (e.g. in Brazilian school) are at least in part explained by a high prevalence of nondirective pedagogical practices. (Soistak, Pinheiro, Galera, 2008). The use of pure memorization methods will have no educational effect, but interconnected with the cultural context, this method may result in great benefits. The use of the Kumon method, for example, may be suitable for children that need to be trained in the procedural aspects of mathematics. The Kumon method introduces the assumption that to achieve any goal, it takes continuous effort and a need to advance step by step without disruptions. Kumon argues that children begin to hate studies when are forced to confront a content beyond their capacity (Kumon, 2001). Considering this, the math anxiety may be associated with an avoidance behavior due to the learning difficulty.

A recent research carried out in public schools of a Brazilian city reinforces the importance of the family to the academic performance. However, the relationship between family and school represents a fundamental aspect for the student learning, having in mind that schools that have the best results not only reproduce the family cultural context, but make an effort to stimulate the best performance of their students (Alves, 2010).

A very powerful strategy to deal more appropriately with the students' individual differences in mathematical learning is to use resources to stimulate the number sense and other primary numerical abilities, based on scientific evidences, specially those from developmental cognitive neuropsychology. The interactions of neuroscience and mathematics education may benefit all children: 1) optimizing the learning of typical achieving children; 2) compensating the deficits of children who present learning difficulties and 3) stimulating pupils who did not have the opportunity to be stimulated in their cultural setting.

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