

The Transformation of Models: from Carpentry to Equations, Airplanes and Computers

A Transformação dos Modelos: da Carpintaria às Equações, Aviões e Computadores

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Abstract

This work consists of a transdisciplinary discussion for engineering training based on the analysis of transformations in the definitions of models from physical objects to computational models. We analyzed changes in mentions of the term “model” in popular dictionaries between the XVIII and XX centuries, finding that definitions for scientific models are more recent than commonly imagined. In parallel, we present the development of differential and integral calculus, from its first concepts to computational use in engineering problems, discussing the role of classical mechanics and the relativistic approach in applied engineering projects. Until the XX century, the definitions of the term model were more associated with models for the manufacture of objects, or artisanal molds typical of everyday life, as well as wooden or metal molds or human models for sculptures or drawings. Definitions of scientific language associated with formalizations, theories or forecasting systems are more associated with the late XIX and early XX centuries. We developed software to use simple differential equations in an engineering application to exemplify the use of applied mathematics in engineering and technology companies and physical models in engineering problems, in order to illustrate the fundamental character of differential equations for problem solving and the importance of Calculus in the training of engineering professionals.

Keywords: Models. Dictionaries. Engineering. Computers. Software.

Resumo

Este trabalho consiste em uma discussão transdisciplinar para a formação em engenharia a partir da análise das transformações nas definições de modelos desde objetos físicos até modelos computacionais. Analisamos as mudanças nas menções ao termo “modelo” em dicionários populares entre os séculos XVIII e XX, constatando que as definições para modelos científicos são mais recentes do que comumente se imagina. Em paralelo, apresentamos o desenvolvimento do cálculo diferencial e integral, desde seus conceitos primeiros até o emprego computacional em problemas de engenharia, discutindo o papel da mecânica clássica e da abordagem relativística em projetos aplicados de engenharia. Até o século XX, as definições do termo modelo estavam mais associadas a modelos para fabricação de objetos, ou moldes artesanais típicos do cotidiano, bem como moldes de madeira ou metal ou modelos humanos para esculturas ou desenhos. Definições da linguagem científica associadas a formalizações, teorias ou sistemas de previsão estão mais associadas ao final do século XIX e início do século XX. Desenvolvemos um software para utilização de equações diferenciais simples em uma aplicação de engenharia para exemplificar o emprego da matemática aplicada em empresas de engenharia e tecnologia e dos modelos físicos em problemas de engenharia, no intuito de ilustrar o caráter fundamental das equações diferenciais para a resolução de problemas e a importância do Cálculo na formação de profissionais de engenharia.

Palavras-chave: Modelos. Dicionários. Engenharia. Computadores. Software.

1 Introduction

The human exercise of understanding the universe involves the composition of ideas to explain how it works, accompanied by formal attempts to synthesize the phenomena. Over time, ideas led to the creation of theoretical models of explanation, as well as criteria for defining models (Lavina, 2004). Models are central in several scientific contexts, since engineers and scientists use them to objectify reality (Kneubil, 2020, p. 1), and, over time, models undergo updates, revisions, and new tests for them to be kept in force (Tonidandel, 2013). Some of the most important models in society were constructed after the development of Calculus, by Newton and Leibniz, because differential equations allowed the calculation of

magnitudes that varied over time, or among themselves. With this mathematical tool, a transformation took place in societies that generated economic and technological consequences.

2 Methodology

The methodology involved the study of differential equations, in the field of function Calculus, consultation of dictionaries and computational software studies. The dictionaries selection was based in five digital collections: (I) the National Library of France (BnF) portal; (II) a British online portal with a listing of dictionaries between the XIX and XXI centuries; (III) the portal of the National Library of Spain, maintained by the Spanish government; the

(IV) portal of the National Library of Germany (Deutsche National Bibliothek, and (V) the Portal of the Digital Library of Munich (Münchener Digitalisierungs Zentrum Digitale Bibliothek). The two questions that guided the article were: (I) is it possible to find a change in the definitions of the term model in dictionaries, including the dimension of scientific models after a certain period of modernity? (II) Is it possible to explain how calculus is applied in real problems and is it also important in computing training?

The notion of model that most influenced the construction of the article was the field of calculus of functions, therefore, it is based on mathematical models. However, it is important to say that there are models and models, especially considering the broad spectrum of scientific areas.

Methodologically, the text did not seek to better detail the difference in models between the sciences. In empirical sciences, such as immunology, models adjust according to experience. In sciences that do not depend so much on experience, models are tested not inductively but deductively by peers. Also, computational models involve database configuration, while mathematical models present more detailed calculations.

3 The Categorization of Models

The categorization of models may involve probing, phenomenological, computational, development models, heuristic models, didactic models, mathematical or formal, mechanistic, instrumental models (Stanford, 2006). Harrison & Treagust (2020) defines a scientific model as “an abstract and simplified representation of a system of phenomena that makes its central characteristics explicit and visible and can be used to generate explanations and predictions” (Harrison & Treagust, 2000).

In a more open sense, rational thinking and planning about a given action already presents the characteristic of a plan or theoretical model. In certain contexts, however, plans must be put to the test to demonstrate their soundness. The existence of criteria allows the evaluation of the legitimacy of a model.

The main models emerged in the XVIII and XIX centuries with the development of classical mechanics, as they were the most powerful constructs for motion analysis. Newton and Leibniz created strong theorems for the study of functions. Derivatives, and later integrals, were developed and united in the fundamental theorem of calculus. Newton's idea was to evaluate the rate of change of a function, which can be used to measure the position of a particle. For this, Newton performed some calculations that produced an application of several works of mathematics with the physical image of world, constant at that time, what changed in a century, with adaptations and probabilities introduced by relativistic problems.

To understand how mathematics connects with physics in unified models, one can start with the idea of the derivative.

The concept of average speed can be seen as $\Delta S / \Delta t = (S(t_1) - S(t_0)) / (t_1 - t_0)$. The instantaneous velocity, $v(t_0)$ appears as a limiting process. We can make a limit as t_1 approaches t_0 , from the position of t_1 as it approaches t_0 , divided by t_1 minus t_0 . At this time, the concept of a derivative is that velocity is the derivative of position. When one knows how to derive, therefore, one can calculate velocities from positions. Considering $v(t)$ = velocity, to calculate the average acceleration, $\Delta V / \Delta t = (V(t_1) - V(t_0)) / (t_1 - t_0)$. The instantaneous acceleration is the derivative and limit as t_1 tends to t_0 . In that regard, $a(t_0)$ is. If we have the velocity function, it can be derived to get the acceleration. From any hourly equation, when differentiating you get the velocity and when differentiating you get the acceleration. If derived again, the rate of change of acceleration can be obtained. With this, any motion can be calculated, not just uniformly varied rectilinear motions.

By the tangent line, it can be observed in the graph of functions that the angular coefficient of a line can be reached by the tangent to a curve of a graph at a point. Being $y = m \cdot x + n$, we have $(y - y_0) / (x - x_0) = m$ (angular coefficient). When there is a graph of x and y with a diagonal line that crosses at least two points, and considering that it has a curve that also passes through the two points, the function can be calculated considering these points and the slope of the line. When looking at the angular coefficient, it is verified that the angular coefficient is the trigonometric tangent of the angle θ . When drawing a secant line, one can see that, at the two points, x_0 and x_1 , when x_1 tends to x_0 , the secant line tends to the tangent, resembling a limit phenomenon. One can take the slope, and then make the secant tend to the tangent. The slope of the secant is the tangent of the angle, opposite leg over adjacent leg. The slope of the tangent line (x_0, y_0) can also be seen as a limit. The rate of change, which is expressed by calculating the derivative, is also present in population growth rates, or rates of change in prices over time (inflation). The number of applications in numerous areas is quite large.

While differential calculus solves the tangent problem, integral calculus solves the area problem by allowing an approximate calculation of the area in a given graph. By the fundamental theorem of calculus, if f is continuous at $[a, b]$, then the function of F can be defined by: $F(x) = \int_a^x f(x) dx$ being continuous on $[a, b]$ and differentiable on (a, b) and: $F'(x) = f(x)$, that is, F is the antiderivative of f . $' = f(x)$ ou d/dx . With integral and differential calculus, several engineering problems could be solved and, as a consequence, different technological constructions took place that provided better business and operational efficiency, as well as new knowledge.

The creation of models has historically gone through a series of epistemological discussions. The first one was: is it possible to obtain knowledge from experience and classify it? This question led to a debate on the problem of induction, started by David Hume, and later addressed by Karl Popper.

The discussion of whether knowledge could be obtained through experience was concluded once it was proven that, positively, the models could be created and tested, as long as their provisional character was assumed, considering the possibility that new, more refined models are presented and accepted (Laux, 2012). The second historical-philosophical debate about the models involved the compatibility of models, starting from the measurement problem, which concerns an unresolved problem of how the collapse of the wave function occurs in the context of debates in mechanics.

Because it was not possible to observe this process directly, different interpretations of quantum mechanics emerged (debate between Margenau, Wigner and Putnam), and a key set of questions were raised that each interpretation should answer. The discussion about the measurement problem was important at a theoretical level for models in science (Pessoa-Junior, 1992, p. 2011). Although this discussion is more specialized, in this article we will address another more superficial approach to models, from their transformation to computational models with the advent of programming.

Not only this works were important for modeling, but also a series of other mathematical works, be it the studies of unknown quantities by Viète, or the analyzes of curves by Euler and Lagrange. The studies of infinite series, as well as studies on quadrature, negative geometries and imaginary quantities in the XIX century (Roque, 2012). The resolution of equations led to the appearance of unknown numbers, in the same way that the “algebraization” of mathematical analyzes were relevant (Roque, 2012).

The advances in mathematics that can be considered as instrumental for the creation of models, involve: theories and analysis of curves, integral and derivative calculus, algebraic methods, studies of infinite series, modification in the concepts of numbers, studies of representations geometry of unknown quantities, studies on functions and limits and in topology. Some of the elements in this set overlap and blend together. Aristotelian logic was translated into set theory in the middle of the XIX century. With this, analytical philosophy was being formed, along with algebras and mathematics. Different sciences contributed to the creation of models.

4 Analysis of the Definitions of the Term “Model” in Dictionaries

The XVIII century was marked by the development of some works in science in the Old World whose bases had been discussed for quite some time. In the work *Discourse on the Method* (1637), by R. Descartes, the basis of rationalism, there was no reference to mathematical models, but in later studies, the term model was better conceptualized. Descartes only mentioned that models served as a reference for comparison in an informal sense: *Que si mon ouvrage m’ayant assez plu, je vous en fais voir ici le modèle, ce n’est pas, pour cela, que je veuille conseiller à personne de l’imiter* (Descartes, 1637). In dictionaries, on the other hand, definitions would

appear, however, scientific definitions would take time to be presented.

In the *Encyclopédie ou Dictionnaire Raisonné Des Sciences, des arts et des métiers*, par une société de gens de lettres, by Diderot and D’Alembert (1750), there are two pages dedicated to the term, being the most complete definition found, probably due to the objective of universalization of knowledge proposed by the encyclopedia in its more than ten constituent volumes (Diderot, D’Alembert, 1750). The main references are to models as molds made of wax or other materials, to models such as naked men posing in drawing classes or models as sculptures or figures in clay. There is no mention of the physical-mathematical sense, although in this period Newton and Leibniz had already created calculus and Galileo had already attested the Solar System with his spyglass (1609).

In the *Dictionary of the French Language (Dictionnaire de la Langue Française)* by Émile Littré (1873), the term “model” refers to models of sculptures or behaviors, but also refers to a representation of a work to be executed (*représentation d’un ouvrage a executor*) (Littré, 1873, p.583). However, the definitions do not cover scientific models (*modèles scientifiques*), or linguistic, or theoretical, or mathematical, physical, or engineering models. In the work *Grand Larrousse de la Langue Française* (1989), by Louis Guilbert, the term modeling does not present an association with scientific models, however, the definition encompasses scientific models: “*modele (...) la science de former des hypotheses, des noms, des modeles*” (Guilbert, 1989, p.3336). Comparison of excerpts from the two works suggests that the scientific perception of models in sampling French dictionaries was not described in 1873, but was reported in 1989.

In the Spanish dictionaries, consulted through the digital collection of the National Library of Spain, it was possible to locate the *Diccionario nuevo de las lenguas, española y french: en que se contiene la explicacion del español en francés, y del francés en español*, by Francisco Sobrino and printed in Brusselas, at the house of book merchant Francisco Foppens. In this work, model was conceptualized as “*dessein de quelque ebose que l’on veut faire*, that is, a drawing sketch of anything you want to do” (Sobrino, 1721).

The definition is particularly interesting, as it starts from a representation created by the individual about something that one intends to create. It is very close to the modeling activity in science, however, the definition was not developed too much so that more elements could be pointed out. But, despite this, the definition is from a work of 1721. Still in the Spanish collection, it was possible to find a version of “*El maestro de las dos lenguas: diccionario español y frances*” by D. Francisco de la Torre y Ocon, printed in Madrid between 1728 and 1731. In this volume, model was conceptualized as: “*modelle, m. Model, bring, plant. item: The man who serves at the Academia de los Pintores for example. Model, imp. Model, rule, agenda*” (Torre & Ocon, 128-1731, p. 260). The

reference to the human anatomy model in art schools also appears in this publication.

Turning to the English works, in the Etymological and Pronunciation Dictionary of the English Language, from 1881, it can be observed that the definition of model refers to models of shapes, or measures, or even to the representation in clay and plastic materials, but not makes reference to the modeling of the outside world in the field of science (Smorthmonth, 1881, p. 364). In Webster's Complete Dictionary of The English Language, published in London in 1886, the term "model" was associated with the definition of planning or forming something from a pattern, a mold, "modeling" being associated with a mold for some work to be done. be executed, like plastic molds (Goodrich, Porter, 1886, p. 848). However, modeling has not been associated with more abstract beings in this book, as a theoretical model. The same occurs with the Chambers Dictionary (1874), which brings more general definitions, at the time of its conception, about the term "modeling", associating it with more everyday situations (Chambers, 1874, p.334).

Turning to the English dictionaries of the early 1900s, in Fowler's The Concise Oxford Dictionary of Current English (1919), the concept of model refers to more popular senses of representation of a structure, in clay, or wood, or plastic or a model for fashion, for clients, but there was no approximation of the mathematical-scientific meaning of the language (Fowler, 1919, p. 522). In this dictionary, measurement (measurement) was defined as the act of: "determining the extent or amount of (thing) by comparison with a fixed unit or with an object of known size; check the size and proportions of (person) to clothes; look (person) up and down with eyes; external mark (line etc. of certain length" (Fowler, 1919).

In the year 1911, a relatively popular dictionary had been published, the Modern Dictionary of the English Language, by MacMillan (1911), in which one can find a definition for the term "measure" which reads as follows: definite unit of capacity or length; the quantity contained in such standard; an instrument to measure; the measurements needed to make an article (like a dress, etc.); (in politics) a purpose, plan or means by which an end is achieved; an act, statute or act of Parliament; a quantity or number contained in another an exact number of times; a dance (in time or measure); (in geology) a series of strata or beds (Macmillan, 1911, p. 418).

The changes in the registers around the concept of model in the dictionaries also related to a slow change of mentalities that occurred in Europe. The gradual critique of Aristotelian conceptions through the absorption of Euclidean geometry, a phenomenon that Alexandre Koyré called "geometrization of space", previously called Aristotelian bases of space, influenced the anthropocentric view of the world of the modern human, giving more power of agency to the human in their abilities to understand the natural world on its own terms. Evidently, several small collapses in cosmological understandings are also reflected in the debate on the idea of

a scientific model, such as Copernicus's own critique of the Ptolemaic system (Sagan, 1995).

During the second half of the 1900s, the mention of modeling as scientific modeling could be found in the dictionary by Aarts, Chalker and Weiner, whose 2014 edition was consulted. The Oxford Dictionary of English Grammar (2014) reported "model" (model) as "An abstract answer or theory of a grammatical or semantic system of a language" (AARS et al, 2014, p. 252). Although the set of dictionaries data is small, it can be observed that, in English dictionaries, the change of records is relatively recent, especially in works that address more general definitions on different topics and that are not dictionaries designed for scientists of a certain area.

Finally, the German dictionaries located in the digital collections of the German National Library and the Munich Digital Library. In German digital collections, it was possible to locate the work Lettisch-deutsches Wörterbuch, by Karl Ulmann (1820), in which there was no definition for model (modell). In the case of the book Deutsches Wörterbuch, by Moritz Heyne (1892), written in Gothic German, the emphasis on models was given in terms of models for construction, for the manufacture of wheels and machines, models of factory products, referring to models French and Italian (Heyne 1892: 845-848). In the Digital Library of Munich it was possible to locate a version of a Franco-German dictionary from 1919, by Karl Sachs, entitled Enzyklopädisches französisch-deutsches und deutsch-französisches Wörterbuch. In this encyclopedia, the term model (modell) was defined as an a priori representation or concept, predecessor to experience, about how an object should be: "le type représente ce que les objets sont aux yeux; le modele montre ce que les objets doivent être; le type est tel que la chose; il faut faire la chose d'après le modele" [type represents what objects are to the eye; the model shows what the objects should be; the type is like the thing; you have to do it according to the model] (SACHS, 1919, p. 183).

In the German dictionary of the XX century, Wörterbuch des althochdeutschen Sprachschatzes, by Gerhard Köbler (1993), no reference to the term model was found in any of its editions. The appearance of the definition of scientific models in dictionaries seems to be due to a sum of factors, from the professionalization of scientific activity, with the emergence of engineering applications, such as the expansion of universities or developments of the Enlightenment, the possibilities of translation and circulation of printed matter, and the advances made by mathematicians, engineers, technicians, workers, commercial and dynamic exchanges in which other local and distant agents participated. We present a history of the development of an equation used in engineering to portray that the appropriation of terms in dictionaries stems from applications of mathematical works.

The process of transformation of concepts is a consequence of the structuring of scientific terms and the development

of inductive method by experimental activity, together with the construction of formal tools and the mathematical constructions. Over time, with the search for symmetries by scientists, it might be possible to identify regularities in nature that led to the transformation of words. In the long duration, what is behind the emergence of the notion of scientific model in popular dictionaries is a significant cultural change, marked by the incorporation of explanations based on heavier empirical evidence and the constitution of engineering organically and the constitution of the field of engineering from the bottom up, in an organic way, taking advantage of the models of classical mechanics and applying them to real problems of companies, businesses, cities, universities.

A good example of how classical mechanical models were used in the construction of engineering problems is the study by Heaviside using the Laplace Transform to electrical circuits. The Laplace transform was a method for solving differential equations that allows the evaluation of the stability and frequency response of a system. Periodic signals, which behave like functions that can be visualized in the time or frequency domain, can be expressed by functions, usually integrals that portray changes in their coverage areas. An example of a modeled signal is the sound pressure over time, while in the frequency domain the different frequencies of musical notes are verified.

Pierre-Simon Laplace (1749-1827) developed an important method for systems, the Laplace transform. Alongside the Fourier transform and the Z transform, they are inputs widely used in signal analysis. The Laplace transform was studied by engineer and mathematician Oliver Heaviside (1850-1926), who demonstrated an application in electrical circuits, published in *The Electrician*. With the transform, one can find the corresponding differential equation, using an operator L as a symbol.

The integral linear operator allows the destruction of derivatives, transforming ordinary differential equations into algebraic equations. The method can be used on piecewise continuous functions, in which the integral converges at certain "s" values, defining a function of s which is called the Laplace transform of s. Considering that "f" is dominated by some exponential, and the exponent of order -st, the function tends to zero over time and, in these cases, the function can be solved by means of elementary formulas. The inverse path, through the inverse transform, can also be performed. With similarities to the Fourier transform, the Laplace transform is expressed by:

$$H(s) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt$$

For continuous-time and impulse-response input x(t) signals, we can consider an output signal y(t) via a convolution integral, such that x(t)=. Since H(s) is a complex constant, we can consider arbitrary values in its formula: □'' + 2□' + 2□ = 10□(□ - 5). In the equation, it is possible to find the second

and first derivatives over time for the function y(t). Derivatives and integrals are calculation concepts with the purpose of measuring quantities, limits, deformations, variations and other parameters in engineering. The previous values in this equation are: y(0)=0 and y'(0)=1. By the conversion properties extracted from tables of Laplace transforms, properties can be obtained:

Property (5):

$$\frac{d^N x(t)}{dt^N} \{f(t)\} \Leftrightarrow SX(s) - x(0^-) \rightarrow sY(s) - 0$$

Property (6):

$$\frac{d^2 x(t)}{dt^2} \Leftrightarrow S^2 X(s) - sx(0^-) - \frac{dx(0^-)}{dt}$$

$$S^2 Y(s) - 1 + 2(SY(s) - 0) + 2Y(s) = 10e^{-5s}$$

$$S^2 Y(s) - 1 + 2SY(s) + 2Y(s) = 10e^{-5s}$$

$$(S + 1)^2 + 1^2 Y(s) + 2SY(s) + 2Y(s) = 10e^{-5s}$$

$$Y(s)(S^2 + 2S + 2) = 1 + 10e^{-5s}$$

$$Y(s) = \frac{1 + 10e^{-5s}}{s^2 + 2s + 2} = \frac{1 + 10e^{-5s}}{s^2 + 2s + 2 + 1 - 1}$$

$$Y(s) = \frac{1 + 10e^{-5s}}{s^2 + 2s + 1 + 1}$$

It is possible to find the poles of equation:

$$1^o = \left(\frac{b}{2}\right) \quad 2^o = \left(\frac{b}{2}\right)^2 \quad 3^o = \left(\frac{\text{function}}{\text{function} + \left(\frac{b}{2}\right) - \left(\frac{b}{2}\right)^2}\right)$$

$$Y(s) = \frac{1 + 10e^{-5s}}{s^2 + 2s + 2}$$

$$Y(s) = \frac{1 + 10e^{-5s}}{(s+1)^2 + 1} = \frac{1}{(s+1)^2 + 1} + \frac{10e^{-5s}}{(s+1)^2 + 1}$$

$$Y(s) = L^{-1}\left\{\frac{1}{(s+1)^2 + 1}\right\} + L^{-1}\left\{\frac{10e^{-5s}}{(s+1)^2 + 1}\right\}$$

$$Y(s) = L^{-1}\left\{\frac{1}{(s+1)^2 + 1}\right\} 10e^{-5s}$$

In this case analyzed by the mathematical procedure, the solution is:

$$Y(s) = e^{-t} \text{sen}(1t)u(t) + 10e^{-(t-5)} \text{sen}(t-s)u(t-s)$$

According to algebraization, the function that describes the electrical circuits can be obtained from bottom to top by the Laplace inverse, considering data obtained by empirical experience. Laplace transformation considers that: , where, F(s) is the Laplace transformed function of f(t); s is a complex variable (can be a complex frequency) and is the base of the

natural logarithm (Euler's number). The Laplace Transform converts a time function $f(t)$ into a complex frequency function $F(s)$.

This transformation is very useful when dealing with differential equations, as it can simplify the resolution of certain problems. It converts a function of time $f(t)$ into a function of complex frequency $F(s)$, which is useful when dealing with differential equations, as it can simplify solving certain problems. It becomes an algebraic equation that is easier to solve. After finding the solution in the Laplace domain, we can apply the inverse Laplace Transform to obtain the solution of the original differential equation in the time domain.

With the improvement of gaps in mathematics and physics, equations and classical mechanics began to solve more complex problems and allow the industrial development of applications in the most different productive sectors. In sub-atomic problems or problems involving time dilation, very high velocities or other characteristics treated with greater precision by theoretical physicists, the calculations of classical mechanics are still valid, but involve more probabilistic issues. On the other hand, quantum mechanics and special relativity are different subjects better analyzed by specialists.

In technological projects of applied engineering that involve real-time control or optimization systems, data are obtained during operation and the systems have internal mathematical functions that correct certain responses of the system itself over time. In some cases, engineering equipment such as embedded systems have functions that are based on calculation principles when they are in operation. With this, mathematics and physics unite with computing to allow important projects for societies to materialize. Airplanes or certain equipment in the health area are concrete examples.

An engineer can build programming codes in which differential equations are used to solve specific engineering problems. If we are interested in modeling the vertical path of an airplane in an ascending or descending flight, we can consider the forces that act on the airplane during its flight, including the lift force, the drag force and the gravitational force. By Newton's Second Law ($F = m.a$) one can describe the vertical movement of the plane as a function of time. The lift force (L) is responsible for lifting the plane and is opposite to the weight force (mg), where m is the mass of the plane and g is the acceleration due to gravity. The drag force (D) is opposite to the direction of motion and is proportional to the square of the plane's speed (v). In this case, the problem consists of solving the differential equation that describes the vertical motion of the plane and calculating its trajectory over time.

In the Python programming language, the SciPy library can be used to solve differential equations, since it has physics frameworks in its content. In the following code, how some input data is collected from the operator beforehand.

The function that uses the library calculates the differential equation without the need for the equation to be solved manually. The differential equation, applied to an engineering problem, brings together knowledge developed mainly from the calculations of Newton and other physicists and mathematicians related throughout the article.

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import solve_ivp
# Function that defines the differential equation of the vertical
motion of the plane
def movimento_aviao(t, y, mass, gravity, lift_coef, drag_coef):
    # y[0] is the position (altitude) of the plane
    # y[1] is the position (altitude) of the plane
    altitude = y[0]
    velocity = y[1]
    # Differential equation for the vertical motion of the plane
    dydt = [velocity,
            (lift_coef * velocity**2 - drag_coef * velocity**2 - mass
             * gravity) / mass]
    return dydt
# Initial conditions
altitude_inicial = float(input("Inform the initial altitude of the
plane in meters: "))
velocidade_inicial = float(input("Inform the initial speed of the
plane in meters per seconds: "))
massa_aviao = float(input("Inform the mass of the plane in
kilograms: "))
aceleracao_gravidade = float(input("Acceleration due to gravity
in meters per second squared: "))
# Lift and drag coefficients or adjust according to the plane and
environment
coeficiente_sustentacao = 0.1
coeficiente_arrasto = 0.05
# Time range for the solution (from 0 to 100 seconds, for example)
tempo_inicial = 0
tempo_final = 100
intervalo_tempo = np.linspace(tempo_inicial, tempo_final, 1000)
# Initial conditions for solving the differential equation
condicoes_iniciais = [altitude_inicial, velocidade_inicial]
# Initial conditions for solving the differential equation
solucao = solve_ivp(movimento_aviao, [tempo_inicial, tempo_
final], condicoes_iniciais,
t_eval=intervalo_tempo, args=(massa_aviao, aceleracao_
gravidade,
coeficiente_sustentacao, coeficiente_arrasto))
# Initial conditions for solving the differential equation
plt.figure(figsize=(8, 6))
plt.plot(solucão.t, solucao.y[0], 'b', label='Altitude do avião')
plt.xlabel('Tempo (s)')
plt.ylabel('Altitude (m)')
plt.legend()
plt.grid()
plt.title('Trajetória Vertical do Avião')
plt.show()
```

Calculus has a significant relevance in the training of professionals who work with engineering. It allows people to see how science somehow connects with reality. Although it has errors, because it is a human language production and does not describe the external reality to the subjects exactly, due to the human limits of reaching the reality itself, it works.

Mathematical modeling of complex aviation systems is necessary for the design, analysis and operation of aircraft,

providing information for engineers and pilots to make safer decision-making at all stages. Differential equations are used to calculate the flight dynamics, in order to describe the movement of the aircraft in three-dimensional space, considering the aerodynamic forces and moments, the propulsion force of the engines and the gravitational forces. In these calculations, it is necessary to model the aircraft's roll, pitch, yaw and altitude changes over time.

In flight control, stability calculations also involve differential equations. Control systems are designed to keep the aircraft on a desired trajectory, stabilize it in different flight conditions, and deal with external disturbances. The equations are also used to model aircraft performance in different phases of flight, such as takeoff, cruise and landing, which includes determining speeds, altitudes, climb and descent rates and distances traveled during different maneuvers.

In aerodynamic calculations, differential equations are also used that govern the flow of air around aircraft, which include equations that describe the behavior of airfoils, drag, lift and other aspects related to force and aerodynamic moment. In systems simulation, differential equations are used to simulate the behavior of complex aircraft systems, such as electrical, hydraulic and control systems, allowing the evaluation of the performance and interaction of systems in different flight situations. Finally, in weather forecasting, differential equations are used in atmospheric models to predict weather conditions around aircraft at different altitudes and positions. These predictions are very relevant for route planning and for the engineering team to decide whether or not to maintain a flight in operation.

In the last century, changes in physics have led to deeper training of engineers in modeling. The possibility of hidden variables beyond the system of balance of forces between two sides brought more "complexity" to classical mechanics. Experimental observations showed that the energy of the electrons depended on the frequency of the light, not the intensity. He proposed that there would be packets of energy, or "quanta", called photons. In this sense, Einstein and Planck realized that the photon behaved like a particle. This generated transformations in particle calculations under certain conditions in the field of engineering.

Engineers consider the field of quantum mechanics in studying the behavior of semiconductors and modern electronic devices, and also in calculating the behavior of subatomic particles on very small scales. Even in aviation, considering gravitational forces, certain concepts of quantum mechanics are taken into account.

When dealing with the uncertainties brought by anti-realism in the construction of complex systems, engineers had to learn to deal with uncertainties brought by the collapse of the wave function in certain situations. Although the data taken from the experiment started to show less consistency, which required more precise calculations and also adjustment systems and a greater need for empirical data for decision

making in relation to certain problems, the corrections that had to be made in the models were small so that satisfactory results could be obtained. The Newtonian/Laplacian clock of the universe remained valid after the need for adjustments to the second scale (Carroll, 2016, p. 166).

5 Considerations

According to Strogatz (2019), "since Newton, mankind has come to realize that the laws of physics are always expressed in the language of differential equations". The consequence of the development of physics, applied mathematics, computing and other areas constitutes critical thinking capable of solving problems associated with variables with dynamic behavior over time. This is of great importance to several segments of the industry. With examples of the context of the use of calculus in engineering, one can better understand its content and how physics and mathematics connect effectively with engineering and computing. With relativistic problems, classical mechanics remains valid but calculations are performed with probabilities.

With the technological advances of the last century, applied mathematical modeling has shown itself not only as a form of epistemological production, but with even a minimal portion of connection with external reality, even if in an "improvised" and temporary way. If the idea of correspondence between logical formulations and external reality itself has been replaced by the idea of a ship that, while moving, the bottom planks are adjusted during its movement, even so the constructs of physics and mathematics remain valid.

In this sense, extreme relativist discourses do not have empirical support, since planes remain in the skies. On the other hand, it must be understood that utilitarian discourses mobilize science and technology as discursive elements. With this article, we seek to carry out a systematic review on the subject of models, from the conception of models as a mould, model, sculpture, behavior, to the theoretical conception of a model, as a paradigm, a formula, a mental, scientific and computational structure. Parallel to this, the visualization of certain calculus ideas and how they make sense in solving engineering computational problems.

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