# QUANTITATIVE STUDY ON TEACHER QUALITY: CASE OF MIDDLE GRADES MATHEMATICS

Mourat Tchoshanov<sup>1</sup>

Mathematical Sciences and Teacher Education University of Texas at El Paso, El Paso, TX 79968, USA

#### **ABSTRACT**

This quantitative study examines whether and how different proxies of teacher quality such as cognitive type of teacher knowledge, coursework and certification are associated with student achievement. In the context of this study, the *cognitive type* of teacher content knowledge refers to the amount and organization of mathematical facts and procedures, concepts and connections, and models and generalizations in the minds of teachers. Teachers were tested using a specifically designed Teacher Content Knowledge Survey. Teacher preparation and teacher demographic characteristics, such as teaching experience, teacher certification, teacher coursework were collected and analyzed with respect to the cognitive type of teacher knowledge and student achievement. The type of teachers' content knowledge was assessed and tested for correlation with student achievement on the state-mandated standardized test using multivariate methods including, but not limited to, tests for variance and independence, and correlation analysis.

**Key words:** Teacher quality; Teacher variables; Cognitive type of teacher content knowledge; Student achievement.

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<sup>1</sup> mouratt@utep.edu

#### THE CASE

Let us consider the following tasks focused on the fraction division. We used similar tasks in the Teacher Content Knowledge Survey to assess cognitive types of teacher content knowledge. How much thinking is required to solve the task below?

Task 1. What is the rule for fraction division?

A. 
$$\frac{a}{b} \div \frac{c}{d} = \frac{ac}{bd}$$

B. 
$$\frac{a}{b} \div \frac{c}{d} = \frac{ab}{cd}$$

C. 
$$\frac{a}{b} \div \frac{c}{d} = \frac{cd}{ab}$$

A. 
$$\frac{a}{b} \div \frac{c}{d} = \frac{ac}{bd}$$
 B.  $\frac{a}{b} \div \frac{c}{d} = \frac{ab}{cd}$  C.  $\frac{a}{b} \div \frac{c}{d} = \frac{cd}{ab}$  D.  $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$ 

One would say, a little or no thinking is required to solve this item. It demands only memorization of the fraction division rule. A pilot study with 22 middle school mathematics teachers showed that 100% of them correctly responded (choice D) to this item.

Task 2 addresses the same mathematical fact – fraction division.

Task 2. Which of the problems below represents the operation  $1\frac{3}{4} \div \frac{1}{2} = ?$ 

- Juan has a piece of rope  $1\frac{3}{4}$  feet long and cuts it in half. At what Α. length should he cut the rope?
- Maria has  $1\frac{3}{4}$  liters of juice. How many  $\frac{1}{2}$  liter containers can she fill? B.
- A boat in a river moves  $1\frac{3}{4}$  miles in 2 hours. What is the boat's C. speed?
- Daniel divides  $1\frac{3}{4}$  pounds of coffee evenly between 2 customers. D. How many pounds of coffee will each customer get?

How different is task 2 from task 1? How much and what kind of thinking is required to solve the task 2? Obviously, task 2 is more cognitively demanding: it requires understanding of the fraction division concept. 72% of the same middle school mathematics teachers (N=22) were able to correctly (choice B) solve this task.

Task 3 builds on the same fraction division fact.

Task 3. Some students mistakenly divide two fractions in the following way:  $\frac{a}{b} \div \frac{c}{d} = \frac{ac}{bd}$ . If a, b, c, and d are positive integers, which of the following holds true:

- A. This equation is always true.
- B. This equation is true when c=d.
- C. This equation is never true.
- D. This equation is true when *ad=bc*.

This task is different from the first two tasks in that it requires respondents to think at a level of *generalization*. Only 41% of the same sample middle school teachers responded correctly to this item (choice B). Not surprisingly, a majority of incorrect responses fell under choice C for task 3. Teachers' performance on tasks 2 and 3, in particular, showed that they lack conceptual and theoretical (generalized) content knowledge on a very basic and fundamental idea of school mathematics – fraction division. We hypothesize that the lack in the type of teachers' knowledge might influence students' learning and achievement.

#### INTRODUCTION

The issue of quality of teacher knowledge and its impact on student achievement is gaining momentum both in research literature and in practice. A recently released report on the Teacher Education and Development Study in Mathematics (TEDS-M, Breaking the Cycle, 2010) emphasizes the idea that "what teachers know and do in the classroom is consequential for students' learning" (p. 4). It seems that teachers concur with this idea. Several middle school teachers who participated in this study were asked to reflect on the following question "Does teacher content knowledge affect student achievement, and if so, how?" Below are some representative excerpts from teachers' written responses:

- "Teachers' knowledge is the main ingredient for students to become great achievers. My students won't learn if I really don't understand what I am teaching" (Bradley<sup>2</sup>, 8<sup>th</sup> grade mathematics teacher).
- "A teacher with more knowledge of the content is able to better explain math concepts to students, in depth and many different ways so that's/ he reaches all students" (Michael, 7<sup>th</sup> grade mathematics teacher).
- "My personal believe [sic] has always been that teachers who have a deeper understanding of content knowledge can be more flexible in delivering the knowledge as well as more capable of breaking down big concepts into smaller connected pieces to ensure student learning and understanding" (Kimberly, 7<sup>th</sup>-8<sup>th</sup> grade mathematics teacher).

It is evident from teachers' responses that they, in one way or another, agree on the importance of teachers' content mathematical knowledge for student success.

What does teacher content knowledge consist of? In our study we use the term *teacher content knowledge* as "the amount and organization of knowledge per se in the mind of teachers" (Shulman, 1986, p. 9), which includes mathematical facts, procedures, concepts, and models as well as why they are true and how mathematical knowledge is generated and structured. Intuitively, we know that if a teacher possesses only knowledge of facts and procedures, then he/she has limited intellectual resources to affect student learning and achievement. Conversely, if a teacher has a profound understanding of mathematical concepts and connections, then he/she might have more opportunities to influence student success. Therefore, in order to detect the specific type of content knowledge, "More precise measures are needed to specify in greater detail the relationship among... middle school teachers' mathematical knowledge... and students' learning" (National Mathematics Advisory Panel Report, 2008, p. xxi).

This study was designed with the following main focus in mind - to measure the cognitive type of middle grades teachers' content knowledge: how well teachers know mathematical facts and procedures (henceforth referred to as Type 1 knowledge); how well teachers understand mathematical concepts and connections

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<sup>&</sup>lt;sup>2</sup> For the purpose of anonymity, teacher names were changed.

(Type 2); and how well teachers acquire mathematical models and generalizations (Type 3). The study looked at the impact of the type of teacher knowledge on students' achievement on state-mandated standardized test and the relationship between teacher content knowledge and other proxies of teacher quality such as coursework and certification. Overall, the study examined the following *research questions:* 

- To what extent are different variables of teacher quality such as teacher certification and coursework associated with teacher content knowledge and student achievement?
- To what extent is the cognitive type of teachers' knowledge correlated with student achievement?
- What is the variance between different teacher rating categories (recognized and non recognized teachers<sup>3</sup>) with respect to type of teachers' content knowledge?

A review of local data in high-stake standardized test shows middle grades students' (grades 5-9) low level of mathematics achievement (percentage of students meeting state standard, 2007), summarized in Table I.

Table I
Student achievement in state-mandated standardized test by grade level

Middle Grades	5 <sup>th</sup> grade	6 <sup>th</sup> grade	7 <sup>th</sup> grade	8 <sup>th</sup> grade	9 <sup>th</sup> grade
Student Passing Rates (%)	85	79	76	71	60

This evidence was one of the major reasons for selection of middle grades for the study: student achievement scores sank in middle grades. In addition, most of the research on teacher content knowledge has been done either at the elementary or high school level.

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<sup>&</sup>lt;sup>3</sup> Following the state accountability system indicators, in our study the term "teacher rating category" is used to define: (1) recognized teachers - with student performance on standardized testing 75% and higher, and (2) non recognized teachers – with student performance at 74% and below.

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The article has several main parts. The first part focuses on theoretical considerations with regard to teacher quality and cognitive types of teacher content knowledge. The second section addresses the methodology of the study. The third part of the paper presents results of the study and its discussion.

#### THEORETICAL CONSIDERATIONS

## **Teacher Quality**

The term "teacher quality' is not well defined in the literature. Scholars use different teacher variables to address teacher quality. For instance, in a study of teacher quality, Rice (2003) analyzes a number of teacher variables with regard to student achievement. Among these variables are teaching experience, teacher coursework, preparation and certification, and teachers' own test scores. Rice concludes that, "Teacher quality matters. In fact, it is the most important schoolrelated factor influencing student achievement" (p. v). Moreover, "No one questions the idea that what a teacher knows is one of the most important influences on what is done in classrooms, and ultimately, on what students learn. However, there is no consensus on what critical knowledge is necessary to ensure that students learn mathematics" (Fennema & Loef Franke, 1992, p. 147). NSF-funded Math and Science Partnership Knowledge Management and Dissemination resource (www.mspkmd.net), based on findings from research, concludes that teacher quality - in general, and teacher content knowledge - in particular, makes a difference in a variety of ways: it influences how teachers engage students with mathematics; it effects how teachers evaluate and use instructional materials; and it is related to students learning and achievement. However, according to the National Mathematics Advisory Panel report (2008), the general results from the existing body of research on teacher knowledge and student achievement are mixed. For example, early studies on the relationship between student learning and teacher knowledge, as measured by standardized tests such as the National Teachers Examination, indicated no significant correlation (General Accounting Office, 1984). Congruently, no important relationships were found between how many mathematics courses teachers had taken and student learning (School Mathematics Study Group, 1972; Eisenberg, 1977; Hill et al., 2005). However, Monk (1994) found that "courses in undergraduate *math education* [emphasis added] contribute more to student gains than do courses in undergraduate math" (as cited in Wilson et al., 2001, p.8). At the same time, Goldhaber and Brewer (2000) found positive effects of teacher certification obtained by high school teachers in a subject-specific domain on student achievement. Several studies conducted by Hill and her collaborators (2004, 2005, 2008) found that elementary teachers' mathematical knowledge for teaching was significantly related to student achievement in first and third grades. The mixed results of previous studies on teacher quality became one of the key motivating factors for designing and executing the proposed project with regard to teacher variables such as:

- type of teacher certification and its association with student achievement
- teacher content coursework and its correlation with student achievement
- teacher knowledge vs. years of teaching experience
- cognitive type of teacher knowledge and student achievement.

The outcomes of the study with respect to these teacher variables are presented in the results section of the paper.

## **Teacher Knowledge and Teacher Proficiency**

In this study we focus on teacher content knowledge, granting that other categories of teacher knowledge (pedagogical content knowledge, for instance) may play a significant role in teacher preparation and effectiveness.

Scholars across the globe emphasize the importance of teacher content knowledge for teaching and student achievement (Hill, Ball, & Schilling, 2008; Davis & Simmt, 2006; Rowland, Huckstep, & Thwaites, 2005; Chinnappan & Lawson, 2005).

Attempts to define teacher content knowledge were intensified in mid 1980s. Shulman (1986, 1987) identified teacher subject matter knowledge along with other categories of teacher knowledge, pedagogical content knowledge, curriculum knowledge, and knowledge of learners, to name a few. Leinhardt and Smith (1985)

defined teacher *subject-matter knowledge* as knowledge which includes "concepts, algorithmic operations, the connections among different algorithmic procedures..." (p. 247). Fennema and Loef Franke (1992) claim that teacher content knowledge "includes teacher knowledge of the concepts, procedures, and problem solving processes..." (p. 162). Hill et al. (2005) proposed that content knowledge could be subdivided into two categories: common content knowledge, or mathematics knowledge that is common to many disciplines, and specialized content knowledge or knowledge specific to teaching. They called the latter mathematics knowledge for teaching and defined it as "the mathematical knowledge used to carry out the work of teaching mathematics" (p. 373).

In the context of this study, the *cognitive type of teacher content knowledge* refers to the amount and organization of mathematical facts and procedures, concepts and connections, models and generalizations in the minds of teachers. Knowledge of facts, rules and procedures in carrying out mathematical processes is called *procedural knowledge*. *Conceptual knowledge* or knowledge of concepts and connections is knowledge about the relationships between mathematical ideas, facts, and procedures. *Generalized knowledge*, or knowledge of models and generalizations, is focused more on adaptive reasoning, logical thinking, and justification. It is well documented that procedural and conceptual knowledge as well as knowledge of models and generalizations are important components of mathematical proficiency (NRC, 2001). A framework for representing connections between teacher content knowledge and teacher mathematical proficiency as mediated by cognitive types of teacher knowledge is shown in Figure 1.

Cognitive types of content knowledge are not hierarchical. For example, if one asks: "Is Type 3 knowledge (knowledge of models and generalizations) higher than Type 2 (knowledge of concepts and connections)?", the answer is "No, they are different". In the study, we had teachers with high Type 3 knowledge and low Type 2 knowledge. Some teachers with more mathematics coursework felt comfortable solving items that required generalizations and less comfortable with conceptual items such as "make up a story for the given fraction division problem".

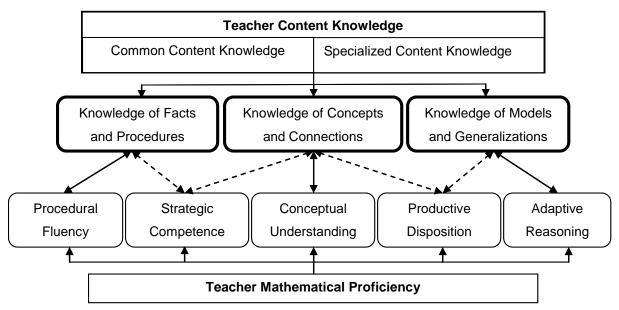


Figure 1. Cognitive types of teacher content knowledge and mathematical proficiency (solid lines indicate strong links, dashed lines – tentative links).

#### **METHODOLOGY**

## Sample Size of Teachers and Students

Prior to conducting the research, a power analysis indicated that at least 32 participants for every subgroup (the estimated mean for a subgroup of recognized teachers was considered at students' standardized test passing rate of 75% and the estimated mean for a subgroup of non recognized teachers – 65%) should be included in order for this study to achieve a minimum power level of .90 with alpha = .01, and an effect size equal or larger than .80. Based on the sample size estimation, 102 subjects were assigned to the study (initially, 105 teachers were tested using the Teacher Content Knowledge Survey; data for one teacher was incomplete and 2 teachers as outliers were removed from further consideration and analysis); 7 teachers out of 102 were first- year teachers for whom the students' scores on standardized testing were not collected, simply because the test administration takes place in the spring of each academic year. Out of the remaining 95 teachers, a subgroup of 62 (>32) teachers was categorized as a group of non

recognized teachers with an average pass rate of students on standardized test at 74% and below, and another subgroup of 33 (>32) teachers – as a group of recognized teachers with students' average pass rate of 75% and above. The main criteria for dividing the teacher sample into two subgroups was the state recognized level of student achievement on standardized test – 75%. The teacher sample represents 12 different middle schools from three major independent school districts in the region, with the population size of 6-8 grades mathematics teachers totaling 463 teachers. The teacher sample size (102) thus represents 22% of the population size.

## **Teacher and Student Sample Characteristics**

Teacher sample demographic information was self-reported by participating teachers and is presented in Table II. The study was conducted in an urban area in the Southwestern USA, consisting of a higher than 70% population of Mexican origin. The teacher sample reflects the demographics of the region: 76% of participating teachers were Hispanics, 16% - Whites, 3% - African-American, and 5% - Other (including, but not limited to, Asians, Middle Easterns, and Pacific-Islanders). In terms of gender distribution, 55% of teacher participants were females and 45% - males. Most of the participants (64%) had 1-5 years of teaching experience. The table below compares teacher sample characteristics to teacher population demographics within the three districts.

Additionally, 62% of the teacher sample received their teaching certificate through traditional teacher preparation programs and 38% of participating teachers were certified through alternative programs. Traditional teacher education program, at the local university, for example, consists of a combination of content and pedagogical coursework, as well as field-based internship at school sites (practicum) as part of a Bachelor's Degree. It takes 36+ credit hours to complete the major professional requirements of the program. In contrast, the alternative certification program is considered a "fast-track" for those who already have Bachelor's Degree in fields other than teaching. 47% of teacher sample were certified as 4-8 Generalist teachers and 53% - as 4-8 Mathematics/Science teachers.

Table II
Teacher sample characteristics

Teacher characteristics	Teacher Sample Data	Teacher Population Data	
Candan	Female – 55%	Female – 66%	
Gender	Male – 45%	Male - 34%	
Ethnicity	White – 16%	White - 24%	
	African-American – 3%	African-American – 2%	
	Hispanic – 76%	Hispanic – 73%	
	Other – 5%	Other – 1%	
Years of teaching experience	1-5 years – 64%	1-5 years – 46%	
	6-10 years – 18%	6-10 years – 20%	
	More than 10 years – 19%	More than 10 years – 34%	

Student sample size was determined based on the teacher sample: the study included about 2,400 students of the participating teachers. Student sample demographics are as following: African American – 2%, White – 5%, Hispanic – 92%, and Other – 1%. 84% of student body in the sample is economically disadvantaged, and 16% are identified as students with Limited English Proficiency. We are cognizant of the impact SES (socio economic status) has on student achievement. However, since the student population was homogeneous in regards to SES, we did not consider it as a separate variable in the study. Claims for the non-biased nature of the study subjects' selection are supported by the evidence of student level data; student performance on standardized test by state, district and campus (average pass rate for students from 12 participating middle school campuses) mirrors each other by objectives as presented in Figure 2.

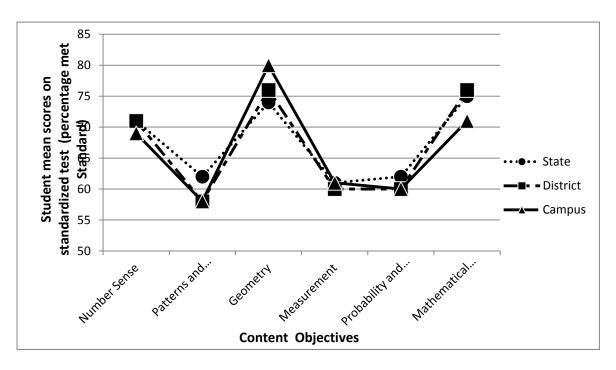


Figure 2. Student performance (average percentage of pass rates) on different objectives of the state standardized test (7<sup>th</sup> grade mathematics, spring 2006 administration) by state, district, and campus.

## Teacher Content Knowledge Survey Reliability and Validity Evidence

In this study, the Teacher Content Knowledge Survey (TCKS) instrument was designed and developed to measure teachers' content knowledge based on different cognitive types. The instrument consisted of 33 multiple-choice items reflecting key standards and competencies for middle grades teachers' knowledge: Number Sense, Algebra, Geometry and Measurement, Probability and Statistics. The instrument development team included interdisciplinary faculty with expertise in the following domains: mathematics, mathematics education, statistics and statistics education, representing multiple institutions: university, community college, and local schools. Item development included the following steps: (1) selection of a test item reflecting a particular standard and competency; (2) identification of the cognitive type to which the item belongs, using descriptors for each cognitive type (Table III); and (3) development of test items that address the same standard and competency for the two other cognitive types.

Table III

Teacher Content Knowledge Cognitive Types' Descriptors

Cognitive Types of Teacher Content Knowledge	Descriptors		
Type 1. Knowledge of Facts and Procedures	<ul> <li>Recognize basic terminology and notation</li> <li>Recall facts</li> <li>State definitions</li> <li>Name properties and rules</li> <li>Do computations</li> <li>Make observations</li> <li>Conduct measurements</li> <li>Simplify and evaluate numerical expressions</li> <li>Solve routine problems</li> <li></li> </ul>		
Type 2. Knowledge of Concepts and Connections	<ul> <li>Select and use appropriate representation</li> <li>Translate between multiple representations</li> <li>Transform within the same representation</li> <li>Transfer knowledge to a new situation</li> <li>Connect two or more concepts</li> <li>Explain and justify solutions to problems</li> <li>Communicate big mathematical ideas</li> <li>Explain findings and results from analysis of data</li> <li>Solve non-routine problems</li> <li></li> </ul>		
Type 3. Knowledge of Models and Generalization	<ul> <li>Generalize patterns</li> <li>Formulate mathematical problems</li> <li>Generate mathematical statements</li> <li>Derive mathematical formulas</li> <li>Make predictions and hypothesize</li> <li>Design mathematical models</li> <li>Extrapolate findings from data analysis</li> <li>Test conjectures</li> <li>Prove statements and theorems</li> <li></li> </ul>		

Test items were screened and revised by a group of experts in mathematics, statistics and mathematics education. The instrument was field-tested during 2005-

2006 (Tchoshanov, Lesser, & Salazar, 2008). The alpha coefficient technique (Cronbach, 1951) was used to assess the reliability of the Teacher Content Knowledge Survey instrument. The value of the coefficient of .839 suggests that the items comprising the TCKS are internally consistent (standard error=.59). The instrument has a respectably high level of reliability, considering the instrument's varying level of difficulty of problems. This is especially significant in light of the report of the National Mathematics Advisory Panel (2008): "Evidence about the relationship of elementary and middle school teachers' mathematical knowledge to students' mathematical achievement remains uneven and has been surprisingly difficult to produce. One important reason has been the lack of valid and reliable measures. The literature has been dominated by the use of proxies for such knowledge, such as certification status and mathematics course work completed" (p. 37).

In order to establish the validity evidence based on test content, a specification table was constructed to guide the process of test development. The table included major content objectives and competencies for teachers. The objectives were closely aligned with corresponding objectives on state-mandated standardized test for students. Aside from the specification table, the item analysis table was used to further ensure validity evidence based on test content. The item analysis table included samples of competencies and items from the Teacher Content Knowledge Survey. These competencies and items were mapped and aligned with competencies and items from the state standardized test for students.

## **Data Collection and Analysis**

The study implemented data collection procedures at two different levels. Teacher level data: measurement of teachers' knowledge was conducted using the TCKS instrument. Each teacher was given 90 min (on average, little less than 3 min per item) to complete the survey and they were allowed to use graphic calculators during the survey. Along with teachers' scores on the TCKS, teachers' demographic information such as: gender and ethnicity, years of teaching experiences, as well as other proxies for teacher content knowledge (i.e., teacher certification and mathematics coursework) was also collected. Teacher level data was collected during 2007-08 academic year.

Student level data: collection of student achievement data was conducted primarily using students' pass scores on the state standardized assessment, which usually takes place in the spring of each academic year. Data were collected from the Spring 2007 administration of the test. To assure accuracy of the data, participating teachers' student pass rates on standardized test were reported by campus mathematics coaches and department chairs.

Data analysis was performed using the following major statistical techniques. Correlation analysis using standard ordinary least square method: the selection of this parametric technique was determined based on the key research question of the study (relationship between teacher characteristics and student achievement), the number and nature of dependent and independent variables as well as the study design and the interval type of scale used for student achievement scores. Non-parametric techniques (test for variance and test for independence) were selected to measure the variance between two independent groups of the same (not normal) distribution with arbitrary sample sizes of each group. The selection of these tests was also based on the ordinal (ranked) data for cognitive types of teacher content knowledge.

#### **RESULTS AND DISCUSSION**

Although measures of teacher quality characteristics in the existing research have varied widely, they are similar in that they focus mostly on identifying categories of teacher knowledge such as: content knowledge, pedagogical knowledge, knowledge of learners, among others. Another commonality in existing research is a focus on the effects of teachers' knowledge on student achievement, primarily in terms of standardized test performance. These commonalities raise the following questions: To what extent is the type of teachers' content knowledge associated with student achievement? Do discrepancies in varied types of teacher content knowledge have different effects on students' success?

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## **Confounding Variables**

The level of complexity and vulnerability of this research - with respect to a number of confounding variables - might intervene in the validity of results of the study. In order to report the results of this study with rigor and accuracy, variables other than teacher characteristics, which may influence student mathematics achievement, were also analyzed. Among these variables were students' achievements in core subjects other than mathematics (i.e., reading, writing, social studies, and science). Student level data from 32 middle school campuses - representing three independent school districts - were collected and analyzed. The most significant correlation was found between student achievement in reading and mathematics, as Pearson's correlation coefficient r=.594, p=.0003<.001. Not surprisingly, the next most significant correlation was found between student achievement in writing and mathematics (r=.384, p=.0003). Surprisingly, the lowest correlation was detected between science and mathematics with Pearson's coefficient r=.191, p=.005.

## Student Achievement's Tendency to Parallel Teacher Knowledge

"It is self-evident that teachers cannot teach what they do not know" (National Mathematics Panel Report, 2008, p. xxi). At the same time, according to Tchoshanov, Lesser, and Salazar (2008), student achievement parallels teacher knowledge. Teachers' performance by TCKS objectives and students' performance on similar standardized test (ST) objectives were analyzed using an average percentage of correct responses. Both teacher and student level data was placed on the same graph to visually represent a revealing pattern (Figure 3). In a sense, it means that if teachers have difficulty in mastering a particular objective then it will negatively impact student achievement on the same objective. Figure 3 shows, for example, that low teacher knowledge on objectives #2 (patterns, relationships, and algebraic thinking) and #4 (measurement) is correlated with low student achievement on the same objectives compare to objectives #1, 3 and 5.

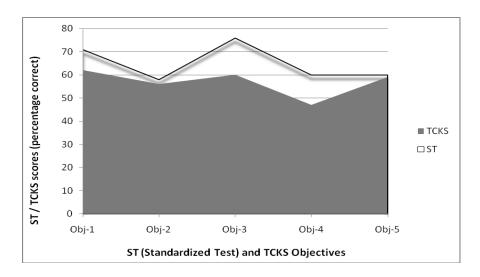


Figure 3. Revealing pattern: student performance parallels teacher knowledge by content objectives (Obj-1 – Number Sense; Obj-2 – Algebraic Reasoning; Obj-3 – Geometric Thinking; Obj-4 – Measurement Concepts; Obj-5 – Probability and Statistics).

We observed a similar pattern in our pilot study (Tchoshanov, 2008) when we calculated means for teachers (N=22) and students' performance by cognitive types of content knowledge. Students' mean scores on standardized test (ST) paralleled teachers' TCKS mean scores, as depicted in Figure 4.

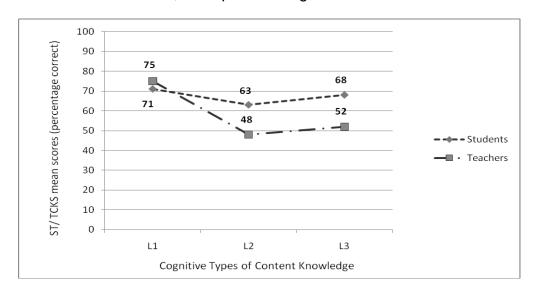


Figure 4. Student performance on standardized test (ST) parallels teacher performance on the TCKS by cognitive types.

One possible explanation of this pattern relates to teachers' reluctance to know mathematics beyond the grade level they teach. An interesting insight

addressing this phenomenon comes from one of the participating teachers who noted that, "As teachers, we need to continually update our mathematical content knowledge beyond the levels which we teach" (Mary, 6-7<sup>th</sup> grade mathematics teacher). Mary had performed low on Type 2 items compared to Type 1 and 3 items of the TCKS that addressed objectives 2 and 4. She further expressed her concern, "... if the teacher has limited content knowledge then the teacher will have a limited amount of ways to introduce key concepts". Figures 3 and 4 capture Mary's concern accurately.

## **Teacher Certification and its Impact on Student Achievement**

Among other teacher characteristics, our study examined the effect of teacher certification on student achievement by program (traditional vs. alternative) and specialization of certification (4-8 Generalist vs. 4-8 Mathematics/Science). The state where the study was conducted offers two types of certification for middle school teachers by specialization:

- 4-8 Generalist certification allows teachers to teach in self-contained classes (mainly in grades 4-6) as well as to teach any subject in grades 6-8, including mathematics.
- 2. 4-8 Mathematics or 4-8 Mathematics/Science certification allows teachers to teach mathematics and/or science only.

"Overall, findings about the impact of teacher certification on student achievement in mathematics have been mixed, even among the most rigorous and highest quality studies" (National Mathematics Advisory Panel, T&TE Report, 2008, p. 3-21). Some studies found positive effects of teacher certification on student achievement (Goldhaber & Brewer, 2000; Rice, 2003) while other studies found no significant effect of teacher certification as a predictor of student achievement in mathematics (Hill et al., 2005; Rowan et al., 2002).

The results of this study confirm the finding that the impact of teacher certification with regard to certification program on student achievement is not significant: students of teachers certified through traditional programs have average test pass scores at 65% and students of teachers certified through alternative

certification programs have average test pass rates at 66%. Moreover, graduates of traditional and alternative certification programs do not differ in terms of cognitive type of teacher content knowledge (observed value of the test for variance F = 1.87 was less than its critical value of 19.0 at p = .348).

## **Teacher Content Coursework and Student Achievement**

Mathematics coursework is another proxy for measuring teacher content knowledge. "It could be that teachers who engage in generous amounts of mathematics course work or obtain mathematics degrees are particularly motivated to teach mathematics, or possess some other unobservable characteristics unrelated to course work and degree that make them especially effective at teaching mathematics" (National Mathematics Panel, T&TE Report, 2008, p. 3-26). Very similar to the research on certification, "the findings in the literature on the impact of content-specific course work and degrees are mixed" (ibid, p. 3-26).

Several studies (Rowan, Chiang, and Miller, 1997; Goldhaber and Brewer, 2000) found that students of teachers with degrees in mathematics perform significantly higher than students of teachers who are not qualified in mathematics. A study by Monk and Rice (1994) used items from the National Assessment of Educational Progress (NAEP) achievement test in mathematics and found a positive relationship between the number of mathematics courses taken and student achievement. On the other hand, a study by Hill et al. (2005) found that teachers' mathematics coursework did not significantly predict gains in student achievement. Moreover, it is less clear "how teachers' level of conventional college mathematics study affects student achievement below ninth grade" (National Mathematics Panel, T&TE Report, 2008, p. 3-27).

The results of this study show insignificant effect of teacher mathematics coursework on student achievement (Pearson's r=.083) despite the fact that a 4-8 Mathematics degree plan (in the public university where the study was conducted) offers 36 credit hours of mathematics coursework compare to only 15 credit hours of subject-specific coursework in a 4-8 Generalist program. Average students' standardized test passing rate of teachers with a 4-8 Mathematics/Science

certification is 66%, in comparison to 62% for teachers who possess 4-8 Generalist certification. At the same time, the study found a variance (non significant though) in the cognitive types of teachers' content knowledge with regard to different certification specializations (F=.841>.052=F-critical, p=.457). It is likely that a variance in depth and scope of teachers' knowledge by certification specialization is due not to Mathematics coursework only, but perhaps to Mathematics Education/Method coursework as well (Table IV). Mathematics Education coursework for 4-8 Generalist degree plan includes only one method class - Teaching Mathematics in Middle School. Along with this course, the 4-8 Mathematics degree plan offers three more math method classes:

- 1) Integration and Alternative Representation of Basic Mathematics Principles
- 2) Introduction to Research in Mathematics Education
- 3) Technology in Mathematics Classroom.

Table IV

Mathematics and Math Method Coursework for 4-8 Generalist and 4-8 Mathematics teacher certification specialization

Specialization	4-8 Generalist	4-8 Mathematics
Mathematics Content	Number of Credit Hours=15 (including Statistics course)	Number of Credit Hours=36 (including Statistics course)
Coursework  Mathematics  Education	Number of Credit Hours=3	Number of Credit Hours=12
Coursework		

This finding concurs with studies claiming that courses in Mathematics Education contribute more to student gains than do content courses in Mathematics (Wilson et al., 2001, p. 8).

## Teacher Knowledge vs. Years of Teaching Experience

Analyzing several studies on the relationship of teaching experience to teacher quality, Rice (2003) concludes that there is a positive effect of experience on teacher

effectiveness; "specifically, the 'learning by doing' effect is most obvious in the early years of teaching" (p. vi) However, our research showed that there were measured differences in content knowledge for teachers (as measured by mean scores on TCKS) with different amount of teaching experience, as depicted in Figure 5. We also observed a phenomenon in which teacher content knowledge "plateaus" in that it comparatively higher for teachers with up to 5 years of teaching experience – at 63% (standard error 2.6 for teachers with 1-2 years of teaching experience, and 2.9 – for 3-5 years), then gradually decreases for teachers with 6-10 years of teaching experience – 60% (standard error 2.8), and continuing thereafter for teachers with 10 and more years of teaching – 55% (standard error 2.7).

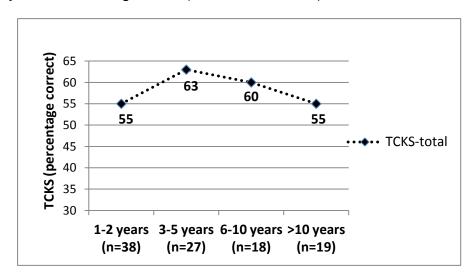


Figure 5. Teacher content knowledge over years of teaching experience.

What specific type of teacher content knowledge contributes most to the difference? When we broke down the content knowledge into types, it appeared that teacher content knowledge at Types 1 and 3 stay comparatively "flat" for groups of teachers with different teaching experiences. Teachers' Type 2 knowledge (mean score) meanwhile, experiences the most distinct drop from a peak of 55% for teachers with 3-5 years of teaching experience, to 43% for teachers with more than 10 years of teaching experience (Figure 6).

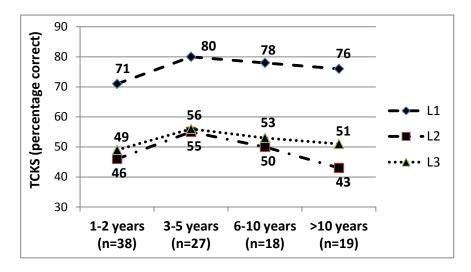


Figure 6. Dynamics of types of teacher content knowledge over teaching experience.

Our observations show that more experienced teachers are reluctant to change as soon as they develop their "comfort zone" and establish traditional teaching routines. This particular outcome of the study supports the notion of ongoing professional development throughout teachers' careers in order to sustain teachers' content knowledge specifically related to concepts and connections.

## Difference in Types of Teacher Knowledge between Recognized and Not Recognized Teachers

Overall, existing research on teacher mathematical knowledge, as measured by a variety of tests, and student achievement, as measured by standardized tests, shows a promising positive trend. For instance, a study by Harbison and Hanushek (1992) found a positive correlation between teacher mathematics test scores and fourth-grade tests on student achievement: "At fourth grade, a ten-point improvement in the mean teacher's command of her mathematics subject matter...would engender a five-point increase in student achievement; this is equivalent to a 10% improvement over the mean scores of fourth graders" (p.114). Clotfelter, Ladd, and Vigdor (2007) examined the relationship of teacher test scores to student mathematics achievement and found that higher teacher test scores are a significant predictor of higher student achievement. Hill et al. (2008) designed a test specifically assessing the mathematical knowledge for teaching and found that the measure of teacher

content knowledge is a significant and positive predictor of student success in mathematics.

As previously noted, in this study 95 teachers were divided into two subgroups. The first subgroup of 62 teachers was categorized as a group of non recognized teachers with average pass rate of students on standardized test at 55%, and the second sub-group of 33 teachers was categorized as a group of recognized teachers with students' average pass rate of 81%. The study results show a difference in type of content knowledge that recognized teachers possess in comparison to non recognized teachers (Figure 7): it seems that Types 1 and 3 of teacher content knowledge are not likely to help us make a clear distinction between recognized and non recognized teachers, at least with regard to student achievement. The data shows that recognized and non recognized teachers have almost the same averages (percentage correct) of Type 1 and Type 3 content knowledge: mean scores of 78% (standard error 2.5) and 75% (standard error 2.7) for Type 1 and 54% (standard error 3.1) and 54% (standard error 3.9) for Type 3 knowledge, respectively. Furthermore, recognized teachers have a significantly high mean score for Type 2 of teacher content knowledge (60%, standard error 3.2) as opposed to non recognized teachers (44%, standard error 2.6).

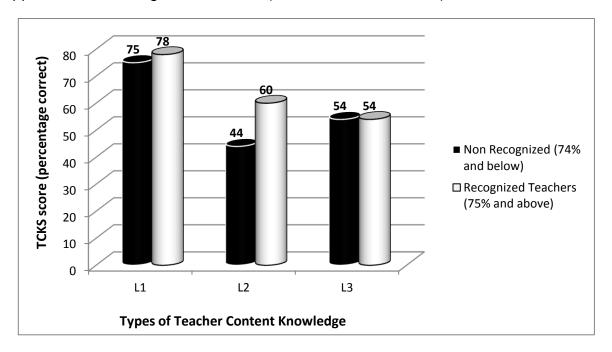


Figure 7. Cognitive types of teacher content knowledge among recognized and non recognized teachers.

This finding suggests a trend; it is likely that Type 2 teacher content knowledge (knowledge of concepts and connections) is more closely associated with student higher achievement. A more dramatic difference in Type 2 knowledge was observed when the teacher sample was further subdivided into teacher rating categories corresponding to the state accountability system (Figure 8) based on student performance on standardized testing:

- Unacceptable teacher performance: average student pass rate (SPR) is below 45%;
- Acceptable teacher performance: SPR is between 45% and 74%;
- Recognized teacher performance: SPR is 75% and above.

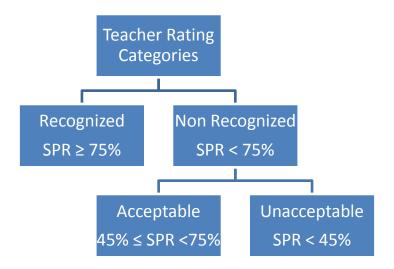


Figure 8. Teacher Rating Categories

The results of the analysis of teacher content knowledge by different rating categories further supported the main claim: the data reflected a minor variation in Types 1 and 3 of teacher content knowledge between different rating categories. However, the variation in Type 2 of teacher content knowledge was significant, as it grows from 42% for teachers with academically unacceptable performance and 46% for teachers with academically acceptable performance, to 60% for teachers with recognized level of performance.

We applied the same teacher rating categories to teachers' performance on the TCKS. The two-way Chi-Square test for independence showed a statistically significant difference between three subgroups of teachers with regard to Type 2 of teacher content knowledge (Chi-Sq = 6.99 > 5.99 [Chi-Sq critical], df = 2, p = .030 < .05). Chi-Square contingency tables along with Chi-Square observed values and p-values for different types of teacher content knowledge are presented in Table V.

Table V

Chi-square contingency tables for different types of teacher content knowledge

Teacher Performance on TCKS	Non Recognized	Recognized	Total		
Type 1 Knowledge	Teachers	Teachers	6		
Unacceptable	6	0	6		
(Percentage correct is below	3.92*	2.08			
44%)	1.109**	2.084	00		
Acceptable	20	13	33		
(Percentage correct is between	21.54	11.46			
45% and 74%)	0.110	0.206			
Recognized	36	20	56		
(Percentage correct is above	36.55	19.45			
75%)	0.008	0.015			
Total	62	33	95		
Chi-Sq = $3.533$ , df = $2$ , p-value = $.1$	71				
Type 2 Knowledge	Non Recognized	Recognized	Total		
	Teachers	Teachers			
Unacceptable	30	7	37		
(Percentage correct is below	24.15*	12.85			
44%)	1.419**	2.665			
Acceptable	24	21	45		
(Percentage correct is between	29.37	15.63			
45% and 74%)	0.981	1.844			
Recognized	8	5	13		
(Percentage correct is above	8.48	4.52			
75%)	0.028	0.052			
Total	62	33	95		
Chi- $Sq = 6.988$ , $df = 2$ , $p$ - $value = .0$	030		•		
Type 3 Knowledge	Non Recognized	Recognized	Total		
,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,	Teachers	Teachers			
Unacceptable	24	12	36		
(Percentage correct is below	23.49*	12.51			
44%)	0.011**	0.020			
Acceptable	27	15	42		
(Percentage correct is between	27.41	14.59			
45% and 74%)	0.006	0.012			
Recognized	11	6	17		
(Percentage correct is above	11.09	5.91	' '		
(1 creenlage correct is above 75%)	0.001	0.002			
Total	62	33	95		
Chi-Sq = .051, df = 2, p-value = .975					
*Expected counts are printed below	ohearvad counts				
		nte			
**Chi-Square contributions are printed below expected counts					

Chi-Square test results show that it is most likely that differences in teacher performance are due to differences in Type 2 of teacher content knowledge - knowledge of concepts and connections.

One of the most important findings of this study was the level of association of the type of teacher content knowledge with student achievement. The relationship between students' pass rates on standardized testing and Types 1 and 3 of teacher content knowledge was not significant (for Type 1 Pearson's r=.06, p=.537, for Type 3 r=.02, p=.853). In contrast, the correlation between students' pass rate and Type 2 of teacher content knowledge was significant with a large effect size (r=.26, p=.009 < .01, r=.89).

The study shows that there is a little evidence for the impact of Type 1 knowledge on student achievement. However, we feel that there is insufficient evidence to claim that Type 3 of teacher content knowledge is not important for student success. One reason for a non significant relationship between Type 3 of teacher knowledge and student achievement might be due to a low number of Type 3 items on the state standardized test. We performed an item analysis of the state standardized test and found that there were only 14% of Type 3 test items compared to 45% and 41% of Type 1 and 2 test items, respectively. Unfortunately, there is a tendency to teach to the test, and some teachers may be following this tendency, thus paying less attention to Type 3 items. Surprisingly, some teachers consider knowledge of models and generalizations "a little above of what the students need to know" at middle grades level (Bradley, 8<sup>th</sup> grade mathematics teacher). Another possible explanation could be that the Type 3 of teacher content knowledge might play a more significant role in student achievement at high school level. Further studies to test this assumption are needed.

## **CONCLUSION**

The research presented in this paper is a focused quantitative study specifically tailored to measure the cognitive type of teachers' content knowledge and its association with student achievement. The study also examined differences in

teacher content knowledge between recognized and not recognized teachers, between teachers certified through various certification routes, and between less and more experienced teachers. The study explored whether teachers' content knowledge of facts and procedures will have different effects on variety of teacher quality characteristics, relative to knowledge of concepts and connections or teacher knowledge of models and generalizations. In order to achieve this research objective, the teacher content knowledge survey instrument (TCKS) to assess types of teachers' content knowledge was developed, validated, and tested, in a number of local middle schools. The results of this study can be summarized through the following categories.

Statistically non-significant findings of the study include the following. The effect of teacher certification with regard to certification program (traditional vs. alternative) on student achievement is not significant. There is no significant difference in Types 1 and 3 of content knowledge between recognized and not recognized teachers. There is a small positive but not significant correlation between teacher content mathematics coursework and student achievement. There is a difference (non-significant though) in the cognitive type of content knowledge for teachers who possess 4-8 Mathematics certification specialization in comparison with teachers certified as 4-8 Generalist teachers. Recognized teachers have higher Type 2 content knowledge as compared to not recognized teachers. There is a positive, but not significant, association of teachers' Type 1 and Type 3 content knowledge with student achievement.

Statistically significant findings of the study consist of the following. Most importantly, a significant association of Type 2 teacher content knowledge with student achievement has been reported. Students' pass rates on standardized test in mathematics and reading are significantly correlated. The study also found that student achievement has a tendency to parallel teacher knowledge. Additionally, teachers' Type 2 knowledge experiences the most distinct drop from a peak of 55% for teachers with 3-5 years of teaching experience to 43% for teachers with more than 10 years of teaching experience.

Results of this study suggest the following trend: teacher knowledge of concepts and connections has a potential to be a good predictor of successful

teachers who positively impact student mathematics achievement in middle grades. However, we are cognizant of the limitation of this study, regarding the narrow focus on teacher *content* knowledge. At the same time, we recognize that the Type 2 of teacher content knowledge "crosses boundaries" of other categories of teacher knowledge, including but not limited to pedagogical content knowledge. It is evident from the teacher responses above that not only teacher content knowledge is important, but that the ability to effectively teach it also matters. Teacher content knowledge, isolated from other categories of teacher knowledge, may not provide a complete picture of a relationship between teacher knowledge and student achievement. Therefore, future studies on integrated teacher knowledge and its impact on student success are needed. One of the major practical implications of this study is that it suggests placing targeted emphasis on the development of teachers' knowledge of concepts and connections while providing content-focused professional development specifically designed to improve middle grades student mathematical achievement.

The outcomes of this research, particularly in the category of statistically significant findings support the National Mathematics Advisory Panel claim that "Teachers' mathematical knowledge is important for students' achievement" but also elaborate on the kind of teacher content knowledge that is critical for student success in middle school. The study shows that teachers' content knowledge of concepts and connections is better associated with students' achievement compared to the teacher content knowledge of facts and procedures or knowledge of models and generalizations.

#### **ACKNOWLEDGEMENTS**

The author thanks Dr. Larry Lesser, Mr. James Salazar, and Mrs. Sherita Martin for collaboration in the teacher content knowledge survey item development, as well as graduate students of his spring 2008 "Research-Based Practices in Mathematics Classroom" for the survey administration and data collection.

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Submitted: October 2010 Accepted: December 2010