THE STRANGE CASE OF THE FOURTH-AND-A-HALF FLOOR: THE WAY 3RD-6TH-GRADERS COPE WITH THE SOLUTION OF A REAL-WORLD PROBLEM

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ABSTRACT

Does mathematics aim to develop logical thinking and find original and interesting relations in its various structures, or to serve as an auxiliary aid for solving problems in areas of science, society and daily life? One could say that the debate is senseless since mathematics is designed for both goals. However, teaching it in elementary school does not take the authentic daily component very seriously, in spite of the curriculum statement of intents. The present study investigated how 3rd-6th-grades solve a real-world problem about the middle floor in a 9-floor building and relate to the result their obtained. The highest success percentage was at the 4th-grade and the lowest at the 6th-grade. Nevertheless, the most interesting finding of the study was the pupils' reference to the illogical and unrealistic answer: the fourth-and-a half floor, being the result of 9:2. About half of the 6th-graders and about one third of the 5th-graders gave such answer without any word of criticism about the obtained number. Only about 13%-15% of the 3rd-4th grade pupils gave such an answer. An inevitable conclusion is that the higher the elementary school age group is, the more the learnt mathematics detaches pupils from reality. The pupils do not use logical considerations, solving problems mechanically, without critical reflection. It is recommended dedicating a considerable part of mathematics lessons at elementary school to the solution of real-world day-by-day problems and to processes of reflection about the received answers.

Key words: real-world problems, thinking, teaching of mathematics, elementary school, problem solution.

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THEORETICAL BACKGROUND

In an article entitled "How old is the shepherd?" Merseth (1993) presents a story problem given to 3rd-graders in the Midwest of the United States: the shepherd has a herd of 125 sheep and 5 dogs. How old is the shepherd? Yes, this is the question and there is no printing mistake. According to Merseth, researchers argue that three out of four pupils will give a numerical answer. She shows an example taken from a pupil's answer sheet: 125+5 = 130, too old! 125-5=120, still old! 125:5=25, that's it, the shepherd is 25 years old.

Merseth (1993) maintains that pupils perceive mathematics as a collection of laws and processes which should be remembered and then applied correctly to the relevant data. The teaching of mathematics in all age-groups directs learners to react with a suitable exercise taken from a familiar algorithmic repertory. There is hardly any guidance and orientation towards critical reflection processes, leading to independent, critical and creative thinking (Gazit 2003).

Unlike Merseth, who discusses elementary school, Bruer (1994) relates to junior high school, reaching a similar conclusion: "Story problems are the black hole of mathematics at junior high school – extensive energy is invested in the content but no light is coming out". One of the reasons for the "black hole" is that most of the problems which pupils at all age-groups are required to solve are not experiential, authentic and relevant to the pupils' world. The problems included in the textbooks engage in a limited area of topics and are presented in a dry and alienated way.

The Israeli poet, Yehuda Amichay (1998), described is well in a concise and creative way in his poem "I remember a question in arithmetic textbook":

I remember a question in arithmetic textbook
About a train departing from one place
And another train departing from another. When will they meet?
And no one asked what would happen when they meet?
Whether they stop or pass one another and maybe clash,

And there was no question about a man departing from one place

And a woman departing from another place. When will they meet?

(free translation, A.G.)

The poem encompasses criticism of mathematical problems, which do not relate to reality. Hence, this gives rise to the following question: What is the essence of mathematics and what is its function among the learnt subjects?

Mathematics is defined as the "queen of sciences" and belongs to science-based subjects, together with physics and chemistry which, in the past, were studied in high school in the "science-based" pathway. However, what is so realistic about mathematics? Mathematics and its various disciplines deal with abstract entities and most of its theorems, principles and processes have no relevance to our daily reality. Yet, it constitutes the basis for the comprehension of principles and laws in various sciences, a basis for understanding physics. Newton, father of the differential-integral arithmetic, promoted mechanisms due to the mathematics he created (not forgetting Leibnitz, who also developed this discipline, grounded on a less applied and more philosophical view). In any case, chemistry is learnt on the basis of mathematics and physics and, based on these three subjects, biology is learnt. One can go on and present psychology as being grounded on the understanding of biology.

An interesting example of that is the work of Piaget (Boden, 1979), who dealt with children's development of thinking. He conceived a detailed theory of the essence of intelligence as a system of constructs created through children's interaction with their environment. In spite of his engagement in and impressive contribution to developmental psychology, Piaget considered himself as a biologist and philosopher of the cognitive development theory. He had a doctorate in biology and his thesis researched sea-living creatures. Piaget was an epistemologist and investigated mainly the development of logical-mathematical thinking. His theory affected the curriculum in mathematics and the methods of teaching the subject.

The basic principle in Piaget's theory is that the source of intelligence is the learner's mutual activity with the environment by assimilation and adaptation processes. The environment is the reality and Piaget recommended interacting with the environment – experiences, illustrations, models. One can also add games and a computer as a basis for cognitive development. However, like many educational theorists and philosophers, in most cases Piaget did not cross the door of the classroom... According to Piaget, children's development periods include the concrete operational stage, between the ages of 7-8 to 11-12. During these years, children learn to perform functions of the first order, such as: combination, arrangement, classification, turning upside-down, relation and so on. Children' thinking no longer needs direct experiencing with an object and they are capable of handling symbols representing concrete situations. Hence, children who are at the concrete stage of thinking can perform introverted thinking functions about realistic situations, being able to combine them with other concrete activities (Boden, 1979).

Nevertheless, not only children have to establish a relation between mathematics and the environment in order to attain a meaningful understanding of mathematical principles. Universities, too, make a distinction between theoretical and applied mathematics, which constitutes a means in various systems, i.e. industry, economy, army, art, sport and other areas.

The mathematician Kronecker said: "God created the integers and all the others are man-made" (Sing, 1997). This theorem can be perceived also as stating that mathematics was created without any relation to reality. However, it can be interpreted as attributing a humanities-based characterization to mathematics, an outcome of human thought. Why, then, is the teaching of mathematics an obstacle and why do many pupils fail to understand it?

However, already at its very beginning as an area of knowledge built on claims, theorems and proofs, starting with Pythagoras' theory of numbers and his famous theorem relating to a right-angled triangle, mathematics engaged in the characteristics of numbers and their geometrical representation, without any relation to their daily application. In Plato's academy, mathematics was studied as a body of knowledge dealing with pure logic. The Euclidean geometry served as a means for developing deductive thinking habits (Steen, 1989).

The geometry that Euclid discussed in his famous book, "The Elements", some 2300 years ago, is studied today with certain changes, introduced only at the end of the 19th-century. Steen (1989) argues that many parents fail to understand why their children need to exhaust their mind with inefficient and uninteresting skills. He recommends focusing on the applications and uses of geometry, analytical geometry and trigonometry in the various scientific disciplines and in our day-by-day life.

One of the objectives of teaching mathematics is using the acquired knowledge in order to solve everyday problems.

Moreover, the new Israeli elementary school curriculum (Ministry of Education, 2007) recommends using situations, taken from our daily life, in the study of mathematics. The American standards for teaching mathematics (NTCM, 2000) also emphasize the need to understand and implement mathematics in our every day life and in our places of work.

Yes, despite the engagement in problem solution during some of the mathematics lessons, textbooks do not relate to the indicated goals or the recommendations. There is a wide gap between mathematics learnt at school and the reality outside the school (Asman & Markovits, 2001).

The focused and pragmatic debate about the objectives of teaching mathematics is usually rare and comprehensive (McNelis & Dunn, 1997). When one asks: "Why do we have to teach mathematics?" the answers are evasive or insignificant. For example: "Everyone needs mathematics". Hogoven (1962) attacks those who claim that mathematics is but an intellectual game. He maintains that mathematics is a useful tool in everyday life and its language is an essential part of the education of members of society.

Russell (1963) expresses a different attitude, according to which mathematics is not a means for the production of machines and weapons, but rather "a cold and esoteric beauty without addressing some part of our nature's weakness". Moreover, he writes that "the true spirit of pleasure and sublimation, the sense of being more than a human being, can be found in mathematics just like in poetry". The comparison with poetry calls for further explanation as to Russell's view of mathematics as a pure science which is not applicable or useful for daily needs.

The debate about the emphasis of beauty and development of thought versus the benefit and application in the teaching of mathematics started, as already mentioned, in ancient Greece. There is a story about one of Euclid's disciplines who asked him about the benefit of teaching geometry. Euclid responded by asking his servant to give this disciple a coin and send him to the street since he was looking only for the benefit in learning (Heat, 1981). Unlike him, Einstein emphasized the importance of mathematics as a powerful tool for use in science studies: "Physics by its very nature is an intuitive and concrete science. It is but a means for expressing the rules which dominate the phenomena" (Calaprice, 1996)

Some mathematics educators concur that teaching this subject should be based on mathematics as a means of communication, as means of organizing the world and the building thereof and a means of handling information (Pimm, 1987). In order to do that, we must change and even eliminate some erroneous beliefs relating to the teaching of mathematics. The most important belief is that teachers and teacher-teachers must stop believing that mathematics is a body of knowledge which is oriented only by rules and is studied by remembering numerical facts and technical calculating processes (Merseth, 1993).

Resnick (1987) maintains that pupils learn at school rules and symbols and when these are detached from reality, difficulties to implement the learnt material might occur. A study conducted in Brazil (Nunes, Schleimann & Caharrer, 1993), found that 9-14 years old boys, who sold products at street booths, made very easily and skillfully various calculations for the purpose of buying and selling. On the other hand, the study examined 6-9 years old pupils, who were unable to apply the mathematics they had learnt for the purpose of calculating prices, they also achieved illogical results. In spite of the methodological limitation of age and development stage differences, one can see a gap between mathematics learn at school and that implemented outside the school. It was Ivan Illich (1973) who wrote in his book about the abolishment of school that children learn everything outside school and in spite of school.

One of the reasons for the gaps between mathematics learnt at school and mathematics required outside the classroom is the uniform "stereotypical" nature of problems studied at school (Gravemeijer, 1997).

Previously, a poem by Yehuda Amichay about a train departing from one place was quoted. Similarly, Russell (1996) criticizes the problems presented in mathematics lessons. She claims that they do not represent reality and that the use of real data is unsuitable and insufficient. She includes an example of a problem whose data are taken from reality: "What is the average length of the seven longest rivers in the world?" wondering, rightly, why it is necessary to know the average length of the rivers.

In addition to the "stereotypical" nature of the problems, there is an additional reason for the difficulty encountered while solving problems in general and authentic problems in particular: problem solution teaching methods which lack processes of systematic and critical reflection.

Polya (1957, 1981) presents four fundamental stages on the way to an efficient and good solution of a mathematical problem: understanding the problem; making a plan for its solution; executing the plan; and reviewing the plan and reflection upon it.

During the first stage, understanding the problem, one should focus on three things: what are we looking for, what are the data and what the conditions are. During the second stage, making a plan, one checks whether pupils have previously solved similar problem or problems and whether they use all the terms embodied in the problem content. During the third stage, executing the plan, one should insert the data into the solution method step by step, phase after phase. During the fourth and last stage, reviewing the plan and reflecting upon it, one should check if the obtained result is logical and if the same result could have been reached also by other way or ways.

The first three stages are implemented to varying degrees during arithmetic lessons. However, the last stage – review and reflection – does not get sufficient attention. This stage of review and reflection is meant to rely on logical considerations as well as acquaintance with the daily reality.

Jacobs and Ambrose (2008-2009) describe a "toolbox", from which teachers can choose means for helping their pupils to solve verbal problems and investigate connections between mathematical ideas. They identified eight categories of the teacher's moves which, at the proper timing, facilitate the identification of the right solution of verbal problems

Asman & Markovits (2001) investigated to what extent daily considerations are manifested when handling problems in mathematics lessons. Their study comprised twenty elementary school mathematics teachers, ten preservice mathematics teachers and fifty 6th-graders. One problem presented to the participants dealt with transportation – dividing 175 passengers among buses, each carrying 40 passengers. How many buses had to be ordered? Another problem engaged in averages: four families live in a building and together they have 10 children. How many children on average does each family have?

All the practicing and preservice teachers answered correctly the transportation problem, explaining their solution based on their acquaintance with the reality of transporting pupils and ordering buses. Some of them even answered they would have tried to put more pupils on each bus so that four rather than five buses could be ordered in order to save money. Unlike them, some of the pupils indicated the answer 4.375, without relating to the fact that it is impossible to order parts of a bus. Among these pupils some wrote "illogical result" while others wrote 4(15), namely 4 buses with a remainder of 15 in order to cope with the issue of fractions. The findings of that study show that pupils solve problems without considering the reality.

As to the problem of the average number of children in the family: 43% of the teachers answered that half a child does not exist and, therefore, they rounded the answer to 2 or 3. They gave this answer although in daily reality, as well as in the communication media, they are exposed to averages which are not integers in discrete values. Several teachers argued that if they had known that an average can be 2.5, then they would have taught it in class. However, textbooks present data in which the average is always a integer. Most of the pupils did not know that it is impossible to obtain an average which is not an integer when we are dealing with the number of people. Low-achieving pupils wrote 2.5 as the result but did it in a technical manner rather than based on their understanding that it is possible to have an average which is not an integer. Conversely, high-achieving pupils answered 2 or 3, like the teachers (Asman & Markovits, 2001) .

The present study explored the ability of 3^{rd}-6^{th} graders (8-11 years old) to cope with an authentic problem which presents real-world data taken from their surrounding environment.

METHODOLOGY

The research objective was to investigate to what extent 3^{rd}-6^{th}-graders use a judgment stemming from daily experiences in order to solve real-world problems.

The research questions were:

1. Will there be any difference in the percentage of correct solutions between the 3^{rd}, 4^{th}, 5^{th} and 6^{th} grades?
2. Will there be any difference in making an incorrect solution – 4.5 – resulting from the rule of division studied at school, between the 3^{rd}, 4^{th}, 5^{th} and 6^{th} grades?

The research population consisted of 189 pupils from the 3^{rd}-6^{th}-grades, learning in a state elementary school at the center of Israel.

The research tool was a questionnaire with the question: what is the middle floor in a 9-floor building?

The research was conducted by means of a page given to the participants, who were requested to indicate the solution and the way by which they had reached the solution.

Findings

Table No. 1 presents the distribution of answers (in %) in the 3^{rd}-6^{th}-grades.

Findings analysis

Table No. 1 illustrates several interesting findings, relating to the research objective and questions.

1. The percentage of pupils answering correctly increases from the 3rd-grade (50.8%) to the 4th-grade (67.7%) – where pupils demonstrated the highest percentage of success! Conversely, in the 5th-grade, the success percentage decreases to 51%, similarly to that of the 3rd-grade (50.8%) whereas in the 6th-grade, the success percentage is the lowest – 36.4%!

Table No. 1: Distribution of answers (in %) in the 3rd-6th-grades

Answer:	5	4.5	4	3	6	9	None	Impossible	4+5	Didn't answer
3rd-grade (N=61)	50.8	14.8	11.5	3.3	8.2		4.9		1.6	4.9
4th-grade (N=62)	67.7	12.9	4.8	4.8	1.6	1.6	4.8			1.6
5th-grade (N=33)	51.5	33.3	3.0					6.1	3.0	3.0
6th-grade (N=33)	36.4	42.4	6.1	3.0* (2.5)			9.1		3.0	
Total	53.9	22.2	6.9	3.2	3.2	0.5	4.7	1.1	1.6	2.6

* One pupil wrote 2.5

2. In parallel to the decrease in correct answers from the 4th-grade to the 6th-grade, there is a considerable rise in the percentage of those giving an unrealistic and illogical answer – fourth-and-a-half floor – from 14.8% and 12.9% in the 3rd- and 4th-grade respectively to 33.3% in the 5th-grade and 42.4% in the 6th-grade!
3. The decrease in the percentage of correct answers from the 4th-grade to the 6th-grade considerably matches the increase in the percentage of the unrealistic answer of 4.5. If we add in each of the grades – the 4th-, 5th- and 6th- – the percentage of correct answers to the percentage of pupils who answered 4.5,

the result would be approximately 80% (80.6%, 84.8% and 78.8% respectively), namely, most of the pupils.

4. What is important regarding the issue of solving a real-world problem between the 3rd- and 4th-grade is the low percentage of pupils whose answer was unrealistic – 4.5 – 14.8% and 2.9% respectively, as compared to the percentage of pupils in the 5th- and 6th-grade who answered 33.3% and 42.4% respectively.

5. If for the 3rd-grade we add the two answers – the correct and the unrealistic - 4.5 – the total is going to be 65.6%. This is a lower percentage in relation to the total of the answers in the three other grades, which is around 80%. About one quarter of the 3rd-graders (23%) gave a wrong answer in integers: 3, 4, 6. However, unlike the answer of 4.5, these floors actually exist.

In the 4th-grade, 12.8% of the pupils gave wrong answers, indicating whole floors: 3, 6, 9, the answer 9 looking like lack of reading comprehension.

In the 5th-grade only one pupil wrote the fourth floor and in the 6th-grade two pupils indicated the fourth floor. One pupil in the 6th-grade wrote 2.5 – an answer which includes half a floor and raises the percentage of respondents with half a floor in the 6th-grade to 45.4%!

6. a. in each of the 3rd-, 5th-grade and 6th-grade, one pupil wrote that there are two middle floors: 4 and 5 (this type of answer is correct for a pair number of floors, i.e. in a 10-floor building).

b. in the 3rd-, 4th- and 6th-grade some pupils wrote that there was no middle floor: 3, 1 and 3 pupils respectively. (This answer too is suitable for a pair number of floors).

c. in the 5th-grade two pupils gave an answer which is, perhaps, similar in meaning to "there is no middle floor", using other words: "Impossible". This answer is also suitable to a pair number of floors. The two word solutions, "impossible" or "there is no such floor", combined with the solution of two middle floors – 4 and 5 – can be added to the unrealistic solution of 4.5, since they attest to a misconception of a real-world situation.

In the 4th-grade, 6.5% of the pupils indicated one of these three solutions; in the 4th-grade – 3.3%, in the 5th-grade – 9.1% whereas in the 6th-grade – 12.1%. If we add up the percentage of these solutions to the unrealistic solution of 4.5, we will obtain in the 3rd-grade – 21.3%, in the 4th-grade – 16.6%, in the 5th-grade – 42.4% and in the 6th--grade – 57.5% (if we also include the pupil who indicated 2.5 as the solution).

A summary of this part of the findings analysis illustrates a sort of an inverted U curve, with an increase in the percentage of correct answers between the 3rd- and 4th-grade. Conversely, between the 4th- and 5th-grade, as well as between the 5th- and 6th-grade, there is a decrease in the percentage of correct answers.

At the same time, there is an increase in the percentage of the unrealistic answer of 4.5: while in the 3rd- and 4th-grade the percentage is similar, it increases in the 5th-grade and continues rising in the 6th-grade.

Solution strategies

Table No. 2 shows the percentage of pupils using various solution strategies.

Table No. 2: Percentage of pupils using various solution strategies

Strategy	Exercise 9:2 9:3	Floor drawing	Points drawing	Numbers in vertical direction	Numbers in horizontal direction	Total
3rd-grade	3.2	3.3	1.6			8.1
4th-grade	3.3 (3x3)	17.7		6.5	1.6	29.1
5th-grade	3.0					3.0
6th-grade	24.2	6.1		3.0	3.0	36.3

Table No. 2 which relates to the solution strategies written by the pupils next to their answer illustrates that:

A. In the 4th-grade, where the percentage of success was the highest, there was also the highest percentage of pupils drawing the floors (17.7%) or indicating numbers which represent floors (8.1%) – about one quarter of the pupils.

In the 6th-grade, where the percentage of success was the lowest, only 12.1% used those same illustrations, whereas about one quarter of the pupils (24.2%) wrote the exercise 9:2.

In the 3rd-grade, two pupils drew the floor and one pupil indicated points representing the floors, in order to reach the correct answer. One pupil wrote the exercise 9:2, leading to the incorrect solution. Due to some reason one pupil wrote that the exercise is 9:3 and his answer was the third floor.

In the 5th-grade none of the pupils used a drawing or indication in order to represent the problem and only one pupil wrote the wrong exercise.

B. The answers included several original and interesting explanations, although some of them led to an incorrect answer:

One 3rd-grader wrote: "There is no middle floor because there is an odd number of floors".

Another 4th-grader gave the correct answer but his explanation was incorrect, "because there is an attic". Hence, it can be assumed that he added the attic to the 9 floors, dividing 10 by 2. The addition of "1" matches the calculation of the middle or the median but he should have added the first floor and not the attic.

One 4th-grader wrote that there is no middle floor, although he drew the floor.

One 5th-grader wrote that a middle floor is not possible because the number cannot be divided. On the other hand, another pupil wrote that there are two middle floors, without indicating their numbers.

A 6th--grader wrote that there is no middle floor because of the odd number of floors while another pupil wrote that you need a pair number. One pupil in that class adopted a different way but his solution was incorrect. When he divided 9 by 2, he obtained 4.5 and erased the half.

DISCUSSION AND CONCLUSIONS

The present study examined to what extent 3rd-6th-graders apply a realistic judgment in order to solve an authentic problem taken from their daily life and their surrounding world (What is the middle floor in a 9-floor building). The first research question aimed to check whether there will be any difference in the percentage of correct solutions between the 3rd- 4th-, 5th- and 6th grades.

The presented problem can be solved in several ways without using calculations: floor drawing, writing signs which represent floors, e.g. point, or indicating the number of floors in order to find out the middle.

Calculation of the middle floor is associated with the process of calculating the mean or the median in statistics. In order to do that one can consider the middle floor as the mean between the first and 0nith floors or view it as a median value of the floor distribution. In both cases the following exercise should be performed: (9+1):2 = 5. When there is no meaningful comprehension of the need to add the first floor, which is not included in the problem data, some of the respondents use the rule of division which is being studied in the range of 1-100 already at the 3rd-grade.

The problem indicates the number of floors – 9 – requesting pupils to calculate the middle, perceived by some of the respondents as division by 2. Perhaps there is here some confusion of terms between middle and half: if we ask what is the middle floor in a 8-floor building, then there is no middle floor. However, if we ask what is the number of half the floors in the building, then the correct exercise is 8:2 and the answer is 4.

In a similar way, one can ask the following question: Two brothers inherited a 9-floor building. Which part of the building will each brother receive? In this case, the answer might be correct in several situations. One option is that the two brothers sell the building and divide the money equally between them. Another option is dividing the apartments on the floor and if their number is pair (2 or 4) – the solution is simple. Regardless of the way for solving the dilemma legally, one can relate to half the building even if the number of its floors is odd.

According to the curriculum, the subject of means is studied at the 5th-grade . Thus, the 5th- and 6th-graders are supposed to use this knowledge in order to solve correctly the problem. However, the problems presented to the pupils relate to the distribution of values, i.e. the amount of milk given by cows in a week or distribution of temperatures per month. When engaging in this type of problems it is clear to the pupils that they have to sum up all the values and divide them by the number of items.

In the problem included in the present study, namely the middle floor in a 9-floor building, pupils are required to apply an analytical critical thinking and reach independently the "hidden" datum – the first floor – in order to add it to the ninth floor and calculate the middle floor.

The percentage of correct solutions given by the 3rd-6th-graders (Table No. 1) indicates an interesting fact which might support the words of Ivan Illich (1973) in his book about the abolishment of school but even more so the conclusion that in arithmetic lessons pupils are taught to make calculations rather than to think.

The 3rd-graders displayed success percentage similar to those of the 5th-graders (50.8% and 51.5%, respectively) and higher than those of the 6th-graders (36.4%)! The highest success percentage was achieved by the 4th-graders (67.7%).

How can we explain this finding, which contradicts our commonsense, expecting an increase in the success percentage during the transition from one grade to another, with the acquisition of accumulated knowledge of mathematical contents. Moreover, the findings illustrate a certain contradition to Piaget's theory: the 3rd-graders (aged 8-9) are at the beginning of the concrete operational thinking stage, whereas the 6th-graders (aged 11-12) are towards the transition to the formal operational thinking stage. It is to be expected that, with age, pupils will increase the percentage of solving real-world problems which combine also formal knowledge, mainly acquired in the 5th- and 6th-grades. However, the findings indicate a decrease with age.

One of the explanation to this phenomenon resides in the distribution of the incorrect and unrealistic answer of 4.5, which constitutes the second research question: Will there be any difference in making an incorrect solution – 4.5 – resulting from the rule of division studied at school, between the 3rd, 4th, 5th and 6th grades?

The lowest percentage in this answer was received in the 3rd- and 4th-grades, 14.8% and 12.9% respectively. In the 5th-grade, 33.3% of the pupils presented this answer whereas the 6th-graders obtained the highest percentage of the unrealistic solution – 42.4%, more than the percentage of correct answers – 36.4!

The 6th-graders, like the 5th-graders, who gave the solution of 4.5, use the division by 2 algorithm for finding the middle, without considering that a floor can be expressed only by an integer. Perhaps, as mentioned before, they also combine immature knowledge which has not reached the stage of meaningful comprehension of calculating a mean, implementing wrongly what they had learnt. Some support for the use of division can be seen in Table No. 2: 22.4% of the 6th-graders wrote the exercise 9:2 as their strategy for solving the problem.

They use the data and execute the plan for solving the problem according to their perception. However, they lack the reflection stage according to Polya (1975), who emphasizes the reviewing and reflection stage, whereby one checks if the solution matches the data presented in the problem. The 3rd- and 4th-graders are perhaps less experienced in division; they have not yet studied the subject of means and the concept of fractions is still at the beginning of its conceptualization. This might partially explain why they used the unrealistic solution very little.

Another possible explanation is that 3rd-graders, as well as 4th-graders, have not yet sufficiently mastered the rules of arithmetic required for solving problems on the one hand. On the other hand, the teaching contents in these grades mainly focus on integers and do not yet include the subject of fractions. This is supported by other incorrect solutions in these grades, manifested by integers – possible floors, although they are incorrect.

In the 3rd-grade, 22.0% of the pupils wrote as the solution the fourth or sixth floors, which are on both sides of the right floor – 5. They also wrote the third floor, although its origin is unclear. In the 4th-grade, 12.8% of the pupils indicated the third, fourth, sixth or ninth floors. In spite of the incorrect solution, it can be assumed that these pupils understand that a floor must be n integer.

More than half of the 6th-graders presented an unrealistic solution, attesting to detachment from the everyday world in which they live. A similar conclusion can be drawn regarding the 5th-graders, albeit in a smaller percentage. Conversely, only a small percentage of the 3rd- and 4th-grades gave an unrealistic solution.

The findings of the present study, with the unrealistic solutions of the 6th-graders, are in line with the findings of Asman and Markovits (2001). In their study, some of the 6th-graders gave an unrealistic answer – 4.375 – to the question about the number of buses required for transporting the pupils.

To sum up the Discussion, one can say with some reservations, that elementary school does not quality its graduates to handle the solution of problems in general and the solution of daily real-world problems in particular. The present study was limited in its scope but constituted a sample of one school at the central area of Israel. The findings thereof illustrate that the learning process in the transition from the 3rd-grade to the 6th-grade, causes learners to be detached from their surrounding world.

About half of the 6th-graders and one third of the 5th-graders consider mathematics as a tool for creating exercises out of numerical data according to patterns which they study mechanically. They do not use critical thinking processes – reflection – in order to check the logic which underlies the solution. The 3rd- and 4th-graders not only gave a higher percentage of correct answers than the 6th-graders; only a few of them gave an unrealistic incorrect answer. They have not yet been "tainted" by the teaching process and are connected to the reality.

An American teacher has always spoken about the failure to integrate historical contents in the teaching of mathematics (the human side of the subject): "... and thus grew the feeling that this entire system (mathematics) had fallen ready-made from heaven for the use of professional jugglers". Perhaps the fourth-and-a-half floor is the delusion of those pupils who actually know that such a floor does not exist. Yet, the teaching of mathematics process puts them into some kind of 'perpetuum mobile": you insert numbers and a technical arithmetic action comes out, in spite of the familiar reality.

It is reasonable to assume that many pupils live in apartment buildings and in the elevators there is not sign for half floors. Perhaps they also wanted to protest against the teaching of mathematics in an alienated, uninteresting and irrelevant way, but this is the subject for another study.

A function of the research conclusions and practical recommendation for the teaching of mathematics at elementary school are to devote more time to the solution of real-world problems taken from the learners' environment as well as to reflection processes, which validate and verify the solution. Presenting problems of this type will not only improve the pupils' problem solution competences but will enhance their pleasure, motivation and interest.

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