THE FUNCTION OF CREATIVITY IN THE SOLUTION OF IRREGULAR SEQUENCE PROBLEMS AMONG 5TH-TO-7TH GRADE STUDENTS AS COMPARED WITH ELEMENTARY SCHOOL MATHEMATICS TEACHERS AND TEACHER TRAINEES IN OTHER DISCIPLINES

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ABSTRACT

This study was conducted following the article "The function of creativity in the solution of irregular sequence problems among elementary school mathematics teachers and teacher trainees in other disciplines" (Gazit & Patkin, 2009). That article examined elementary mathematics teachers and teacher-trainees ability to solve irregular challenging problems of sequences in general rather than numerical sequences only. The present article aims to present the results of a study examining the function of creativity in solving irregular sequence problems in 5th-7th grade students at a democratic school, and in comparison with the Gazit and Patkin (*ibid.*) study population. The findings show that the more the question deals with a less-common sequence and demands open thinking associated with creative thinking, the lower the level of coping. Another and almost surprising finding show that the ability of students in grades 5-7 to solve irregular sequence problems is no lesser than that of teachers and teacher trainees. An obvious conclusion is the need to deepen and broaden the teaching of irregular-problem solving at all school age levels, including teacher trainee and to devote time for developing creative thinking.

Keywords: Creativity, problem solution, mathematics, students, 5th-7th grade, irregular problem, sequences.

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INTRODUCTION

Mathematician Karl Weierstrass said that a mathematician who is not also something of poet will never be a complete mathematician. An attempt to unlock the inherent beauty of math and poetry reveals that they share certain modes of thinking. A poem uses a handful of words, as math uses a handful of words and symbols, to express an entire world (Gazit, 2001).

In his book *A Mathematician's Apology*, mathematician G.H. Hardy maintains that:

> *"The mathematician's patterns, like the painter's or the poet's, must be beautiful." (Hardy, 1941, p85)*

Mathematician and philosopher Bertrand Russell adds that we can find the true spirit of joy and transcendence in math as much as in poetry (Monk, 1997). Adding to these mathematician Kroneker's statement that: "God made natural numbers. All the rest made by mankind" (Sing, 1997), we have cause to presume a need for fostering creative thinking as part of math instruction. Dealing with sequences gives opportunities for elaborating creative processes.

The purpose of the present study is to investigate the ability of students in grades 5-7 to deal with the completion of irregular sequences, not necessarily numerical sequences.

THEORETICAL BACKGROUND

Problems vs. irregular problems in math

A problem is composed of a verbal situation containing various data. The problem is normally drawn from everyday life and concerns mathematical objects such as numbers and shapes in repetitive structures. To solve a problem, students must represent the situation and the data in a mathematical model familiar to them.

An irregular problem enables to test the application of study matter at levels beyond algorithmic reconstruction or procedures rehearsed in class. According to

Gazit (2003), using riddles and thinking challenges in school will not only benefit the development of thinking skills, but also increase motivation and interest among students of all levels.

In their article, Clark and Clark (2003) present beliefs of high school students in grades 9 to 12 regarding the subject of mathematics, according to a study by Schoenfeld (*ibid.*):

a. Math problems posses only a single correct answer.

b. There is only a single method for solving any math problem.

c. Ordinary students cannot be expected to understand math.

d. Math is an individually performed activity.

e. Every math problem can be solved in five minutes or less.

According to Clark and Clark, such thought patterns most likely begin solidifying in the early years of elementary school. They believe that the use of challenging, fascinating problems can affect children's beliefs regarding math and regarding themselves as learners. Their article presents the experiences of 2nd graders in dealing with two irregular problems within a supportive and challenging environment. The students reached solutions that were both original and correct, and derived great satisfaction from their efforts despite the problems being rather difficult. Solving irregular problem with original strategies has a sense of creativity, which is an important cognitive tool in mathematics inquiry.

Creative problem solving

Similarly to Polya (1957), who delineates four stages for solving a mathematical problem: understanding the problem; devising a plan; carrying out the plan; evaluation and critique, Butcher (1968) also describes the creative process in four stages:

a. Preparation – collecting information, and attempting to reach a solution using standard methods.

b. Incubation– a "hidden" stage featuring unconscious thought.

c. Illumination– the stage at which an idea suddenly emerges as if through inspiration. This stage is normally spontaneous and unlabored.

d. Verification – the stage at which the solution is tested and processed.

The two middle stages –Incubation and illumination, are not evident or obvious. It is unclear how the idea "pops up" in those two stages, and what unconscious thinking takes place.

The ability to attain creative thinking incorporates high thought functions Resnick (1987) lists the characteristics of learning activities that incorporate high cognitive functions:

a. Complex cognitive activity.

b. Non-algorithmic cognitive activity.

c. Cognitive activity that enables to find several solutions.

d. Cognitive activity that requires the use of different criteria.

e. Cognitive activity involving uncertainty.

f. Cognitive activity requiring self-regulation and self-direction.

g. A cognitive process that creates meaning.

h. An activity requiring mental effort.

Creative problem-solving in mathematics

Creativity in mathematics is manifested by the independent articulation of uncomplicated math problems, finding methods and means for solving these problems, and finding original methods for solving irregular problems. One method for creating situations that require creative thinking is to present open problems where there is no single, unequivocal solution. If we ask students how to equally divide 12 apples into 3 bowls, the algorithm is unequivocal and the answer under the given conditions is single. However, if we ask how to equally divide 12 apples into *several* bowls, there is no one solution, and the student must make assumptions before providing the solution from among several possible answers (Yee, 2005).

161 – v.5(1)-2012

JIEEM – Jornal Internacional de Estudos em Educação Matemática
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Solving irregular sequences is an example of task which involves many ways to get the answer and in some cases- no one solution.

Irregular sequences

Solving regular sequences such as arithmetic or geometric has standard algorithm to be use, sometime, in different ways. Irregular sequenced have no a priory algorithm and we need to find the pattern by means of divergence thinking. Students in the secondary school have some beliefs about the nature of sequences (Przenioslo, 2006). These beliefs include ideas such as : 1. Sequence must be monotonic. 2. The difference between terms must be the same. 3. The terms must form a formula. 4. There must be some harmony. 5. A sequence is a list of numbers. 6. A sequence must be regular. The author recommended introducing irregular sequences in which the pattern is "hidden" as to promote more understanding of sequences, parallel to creative competences.

Child and adult creativity

In order to distinguish between child creativity and adult creativity, we must determine the manner in which knowledge exists in us. According to Gilda (2004), we have a set of patterns, we identify things by categorizing them, and we link between things using an orientation map. For example: we identify a certain piece of furniture as a table mainly due to the horizontal plain located at a certain height, sometimes higher than the chairs next to it. Knowledge is thus expressed in the ability to organize patterns, store them, and recall them in an appropriate context. In this regard, a creative idea is a new pattern, or the updating of an existing pattern.

Thus, what is the distinction between child creativity and adult creativity? Children's ability to update an existing pattern is lesser than that of adults, yet their curiosity is higher on average than that of adults. Therefore, Gilda (2004) asserts that children are not more creative than adults, but rather posses a different type of creativity – *childlike creativity*. This creativity is based on lack of knowledge and of experience. Adult creativity is based on alternative and on an updating of patterns.

That is to say, we have multiple alternatives for understanding and acting regarding every problem. Belief in additional alternatives drives us to keep on seeking, and eventually also to find, new ideas.

Gilda (2004) goes on to claim that things must be thought and expressed in simple fashion, and that there is no need to be overly clever and think "differently" in terms of complex, complicated thinking. Gilda maintains that in most people there is a clear correlation between creative ability and simplicity of thinking. In the Gazit & Patkin study (2009), two adults groups completed a battery of sequences of different irregularity. One group was of elementary mathematics teachers and one group was of teacher trainees from other disciplines. The finding showed that the extent of success was associated with the extent to which the sequence was deviated from familiar patterns. The sequence with the lowest success was the one with the sequence of the letters representing the initials of the week's days (Third sequence, appendix 1). Although, the teacher trainees none mathematics presented less success in comparison with elementary mathematics teacher, their incorrect answers included creative irregular notions. (Gazit & Patkin, 2009)

Research questions

1. Is there any difference in the solution of irregular problems between numerical sequences and non-numerical sequences, e.g., sequences of letters or geometric shapes?

2. What are the differences in the solution of irregular problems between 5th-to-7th graders and adults – the teachers and teacher trainees examined in the Gazit and Patkin (2009) study?

METHODOLOGY

A comparative, descriptive study in which subjects of three graders groups answered a sequence-completion questionnaire (Appendix 1). The percentage of the

correct answers was calculated and compared to the percentage of the correct answers in previous research (Gazit & Patkin 2009)

Study population

80 students grades 5-7 from the Democratic School in central Israel who took part in math and geometry classes during the 2009-2010 school year.

- In the 5th grade 29 students took part (18 boys and 11 girls).

- In the 6th grade 24 students took part (14 boys and 10 girls).

- In the 7th grade 27 students took part (15 boys and 12 girls).

- In total, 80 students took part (47 boys and 33 girls).

At the **Democratic School**, children are free to choose how they wish to spend their school hours: in class, at a learning center, in a workshop, at play, at crafts, in conversation, and by strolling through the school yard. This is based on the belief that by experiencing choice processes the students will learn to know themselves, what they love, and their inclinations, and thus reach maximum potential and self-realization. The school offers students a choice of their curriculum. The timetable includes learning hours that are courses and learning hours where individual children may study either sporadically or permanently at the learning centers. Every student has personal tutor who are involved in the student's learning, cultural and personal development. The mathematics instruction is base on student's choice as well as on dialog between the student and his tutor. Emphasize is given on the learning process and not on selective materials and textbooks. Many kinds of curricular materials are available to give response to the heterogeneous in students learning styles and competences.

Thus, not all students in grades 5-7 participated, but only those who elected to study math and geometry in the 2009-2010 school year.

164 – v.5(1)-2012

JIEEM – Jornal Internacional de Estudos em Educação Matemática
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Research tools

A work page containing 5 irregular challenging questions on sequences (Gazit and Patkin, 2009).

The students were asked to complete the next member (a_{n+1}) in each of the sequences (Appendix 1).

FINDINGS

Differences in success rates between the different sequences

Table 1: Answer distribution in percents by type of sequence (numerical sequences and other sequences)

	Correct answers	Incorrect answers	Missing answers
Sequence 1	95.0%	5.0%	0.0%
Sequence 2	65.0%	27.5%	7.5%
Sequence 3	17.5%	45.0%	37.5%
Sequence 4	36.3%	40.0%	23.8%
Sequence 5	73.8%	7.5%	18.8%

Table 1 shows that Sequence 1, which deals with numbers, garnered the highest percentage of correct answers (95.0%); followed by Sequence 5, which deals with geometric shapes (73.8%); then Sequence 2, which deals with numbers (65.0%); then Sequence 4, which deals with numbers and words (36.3%); and finally Sequence 3, dealing with letters, which received the lowest percentage of correct answers (17.5%).

The findings indicate significant differences among the various sequences, with the exception of sequences 2 (numbers) and 5 (shapes). The sequences dealing with numbers and shapes received a success rate ranging between 65% and 95% - most subjects answered them correctly. In contrast, the sequences dealing with letters and shapes with words received a success rate of between 17.5% and 36.3% - most subjects answered them incorrectly.

Differences between 5th-7th grade students and adults (teachers and teacher trainees)

Table 2: Distribution of the students' answers compared with the adults' answers in Gazit, Patkin (2009)

		Correct answers	Incorrect answers	Missing answers
Sequence 1	Students	95.0%	5.0%	0.0%
	Adults	94.1%	5.9%	0.0%
Sequence 2	Students	65.0%	27.5%	7.5%
	Adults	52.9%	35.3%	11.8%
Sequence 3	Students	17.5%	45.0%	37.5%
	Adults	13.7%	64.7%	21.6%
Sequence 4	Students	36.3%	40.0%	23.8%
	Adults	52.9%	27.5%	19.6%
Sequence 5	Students	73.8%	7.5%	18.8%
	Adults	88.2%	7.8%	3.9%

Table 2 shows the percentage of correct answers for the students, comparing with adults. The average success of the five sequences is slightly higher within adults (60.4%) than within students (57.5%), by a negligible difference. Globally, one can say that 5th-7th grade students manage irregular sequence problems similarly to adults – elementary school math teachers and elementary school math teacher trainees.

Sequence 1

There is almost no difference between the students and the adults, with 95% of the students and 94.1% of the adults answering correctly. These percentages constitute the highest correct answer rate from the five sequences, both in students and in adults.

Sequence 2

The students' correct answer rate (65.0%) is higher than that of the adults (52.9%) – a sizeable and rather surprising difference. It is possible that the students,

unhindered by preconceived schemes and patterns, manage to seek unconventional models, while the teachers and teacher trainees seek a familiar algorithm.

Sequence 3

There is a slight difference between the students and the adults, with 17.5% of the students and 13.7% of the adults answering correctly. These percentages constitute the lowest correct answer rate from among the five sequences, both in students and in adults. But, investigating the incorrect answers, we found some creative completion among the students answers. The sequence to be completed: r, s, s, r, h, s…. which are the initials of the six days of the week in Hebrew. The seventh letter is s (for "Sabath) but some students did not follow the instructions for only one letter an answered: s, h. It is creative because they found a pattern of 1,2,2,1 and repeated it.

Sequence 4

The students' correct answer rate (36.3%) is lower than that of the adults (52.9%). This may be partly attributed to the students' insufficient knowledge on geometrical shapes at this point in their studies. This is also the second most difficult sequence after the letter sequence (2).

Sequence 5

In this sequence, the students' correct answer rate (73.8%) is also lower than that of the adults (88.2%), a finding that may support the reason presented in Sequence 4, which also deals with geometric shapes.

DISCUSSION AND CONCLUSIONS

The purpose of the study was to investigate the ability of students in grades 5-7 to deal with irregular thinking challenges in the field of sequences, some of which were non-numerical. These problems have no preset algorithm, and require some degree of breaking regular thinking patterns and switching towards creative thinking characteristics: seeking a new and unknown model of solution from a data sequence that presents familiar items, yet at a context not known in advance. The subject must

make independent assumptions without preconceptions before selecting the appropriate solution to the open question (Yee, 2005).

The first research question examined whether there are differences between numerical sequences and other sequences. The findings in Table 1 indicate that there are differences in the percentage of correct answers between problems presenting numerical sequences and problems presenting non-numerical sequences, such as letter sequences, or geometric shape sequences combining names of the shapes. These finding support the findings of Przenioslo(2006) which indicated that secondary students have some fix conceptions about sequences. These conceptions include the beliefs that sequences must be numeric and regular. Mathematical thinking is normally identified with numbers and shapes, while language, which constitutes a medium for instructions and questions, is perceived as separate from the actual question data. In sequences, however, study materials normally address only numerical sequences, while sequences dealing with shapes or letters are not part of the math study routine. The problem that scored the lowest success rates was the one that presented a letter sequence representing the prefixes of the weekdays (Sequence 3). This question was different in content from the other questions, as it dealt with letters rather than numbers and shapes. We can deduce from this that a question with data to which one is unaccustomed from one's school studies – letters rather than numbers – creates a thinking block and prevents proper coping.

A problem with low success rates, though higher than those of the letter sequence, was the problem that combined geometric shapes with the shapes' names (Sequence 4). The subjects had to identify a model of ascending alphabetical order and only few more than a third of the subjects answered correctly. This reinforces the conclusion that combining unaccustomed elements in standard problems, also in relation to sequences, disrupts regular thinking at certain norms of data and data-interrelations.

We may say with a fair degree of confidence that a difference was indeed found between numerical or geometric shape sequences and non-numerical sequences such as those with letters or those combining geometric shapes and words. This difference indicates a sort of mental fixation: with instructions for completing a numerical sequence there is only a single number; but when a shape

with a name must be selected, there are multiple possibilities, as well as difficulty in finding the model.

The third most difficult problem did deal with a numerical sequence, but not a regular one. The problem presented a number sequence (Sequence 2) in which each number equals the sum of the two previous numbers. This sequence takes after the famous Fibonacci sequence, and such a relation among numbers in a sequence is not regular. Regular sequences are characterized by a fixed model of numerical difference, proportion, or difference of differences, with the model being visible on first sighting. The model of adding two previous numbers enables to create multiple sequences, and its structural logic is visible. However, as it is not regular, there is "blindness" – a fixation. Despite this, about two thirds of the students succeeded in answering this sequence correctly.

The problem with the second highest success rate after problem no. 1 was the problem dealing with a sequence of polygons whose number of sides decreases by two from one shape to the next (sequence 5). The model is very clear, and the problem in fact restates a numerical arithmetic sequence using shapes.

As noted, sequence 1, which dealt with differences, had the highest success rate of the five sequences – despite the fact that the 5th-7th graders do not formally study about sequences.

We can say that the more the question deals with a less-common sequence and demands open thinking associated with creative thinking, the lower the level of coping. The success rate, which ranged between 95.0% and 73.8% for more common problems, drops to 65% in a less common problem, down to a low success rate of 36.3%, and to the near-failure of 17.5% for problems venturing beyond classical math (numbers and shapes) and presenting a sequence of letters related to the days of the week, or shapes with their names – supposedly familiar situations, yet in an uncommon context. Solving these types of problems puts a premium on creative thinking – breaking normal thinking patterns and finding new directions and original methods of solution.

The second research question examined differences between how the students coped in this study and how adults coped in the Gazit and Patkin (2009) study. No significant difference was found between the 5th-7th grade students and the

adults – the elementary math teachers and teacher trainees. In task 2 – the Fibonacci-type sequence – the students scored significantly higher, while in sequences 4 and 5, which dealt with geometric shapes, the adults had a somewhat significant advantage. Fibonacci sequence is a very famous one and mentioned as enrichment in mathematics class. In the Gazit & Patkin study (2009) there was a significant difference between mathematics teacher (73%) and pre-service none mathematics teachers (44%), responding the Fibonacci-type sequence. The percentages of the students (65%) are closer to that of the teachers and differ significantly from that of the pre-service teachers. An alternative explanation to the contradict findings is that the students and the teachers are aware to Fibonacci sequence while adults-pre service teachers not mathematics, who are not connected to mathematics phenomena, have not the clue to the answer.

In summary, we can say that the less routine the sequence, and the more it requires creative thinking, the fewer the correct answers. This can be primarily attributed to the fact that the school does not teach enough problem-solving strategies for students to use in solving irregular problems, such as sequences. An obvious conclusion is the need to deepen and broaden the teaching of irregular-problem solving at all school age levels, and to devote time for developing creative thinking. This conclusion is identical to the conclusion of the teacher and teacher trainee study (*ibid.*, 2009), reinforced by the findings of this study, which show that the ability of students in grades 5-7 to solve irregular sequence problems is no lesser than that of teachers and teacher trainees.

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171 – v.5(1)-2012

JIEEM – Jornal Internacional de Estudos em Educação Matemática
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APPENDIX 1

Complete the next member in each of the sequences.

Explain your choice – if needed.

You may write words or draw

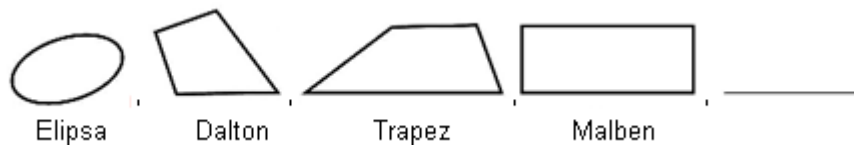**First sequence**   1, 3, 7, 15, 31, _____

Second sequence   1, 3, 4, 7, 11, _____

Third sequence   r, s, s, r, h, s, _____

 * (These are the first letters of the weekdays in Hebrew)

Fourth sequence

Elipsa   Dalton   Trapez   Malben

 **(Here the Hebrew shape names are ordered alphabetically)

Fifth sequence

*The letters represent the first letter of the day of the week in the Hebrew alphabet :

Sunday (Rishon), Monday (Sheni), Tuesday (Shlishi), Wednesday (Revi'i), Thursday (Hamishi), Friday (Shishi)

**The geometrical shapes are arranged according to the first letter of their name in Hebrew alphabet: ellipse (ellipse -1st letter in the Hebrew alphabet), kite-shaped quadrangle (dalton – 4th letter), trapeze (trapeze – 9th letter), rectangular (malben- 13th letter)