WORKING MEMORY AND MATHEMATICAL THINKING: A COGNITIVE AND AFFECTIVE NEUROSCIENCE APPROACH

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ABSTRACT

Good performance on mathematical tasks requires the development of highly complex skills. The working memory model provides a useful framework for understanding the role of the different cognitive mechanisms involved in these mathematical skills. The present paper reviews several neuropsychological and neuroimaging studies, suggesting that mathematical performance depends on working memory resources. The paper begins with a description of the different working memory components. We then present evidences that suggest that each working memory component plays a crucial role in mathematical problem solving. We also review numerous studies that show that working memory and mathematical thinking share a considerable number of neural circuitries within the posterior parietal cortex and prefrontal regions. Finally, we discuss how anxiety might jeopardize working memory capacity and thus reduce performance in solving mathematical problems. Measures that increase working memory capacity might improve mathematical problem-solving achievement. Interventions based on controlling the

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negative feelings that precede mathematical performance might also be helpful for people who suffer from mathematical anxiety.

Keywords: number sense, number magnitude, neural circuitry, anxiety, working memory training.

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INTRODUCTION

How much is 37 times 7? Before you continue reading, try to solve this mathematical problem. Did you reach a result? If your answer was 259, then congratulations! You are right. Now try to describe the mental operations that led you to the correct answer. Most likely, you first multiplied 7 by 7, which equals 49. Then you probably kept the 9 as part of the final answer and added 4 to the result of multiplying 7 by 3, which equals 25. You then came to the result of 259. This simple mathematical problem required the performance of several mental functions that might be associated with working memory. The purpose of the present paper is to provide a brief literature overview on the relationship between working memory and mathematical ability, such as number sense, number magnitude, and basic arithmetic algebraic operations. We begin this review by pointing out some of the historical aspects of the concept of short-term memory that resulted in the development of the working memory model proposed by Baddeley and Hitch (1974) and updated by Baddeley (2000).

WORKING MEMORY MODEL

The idea that human mental faculties have a temporary memory system was already present in the thinking of the ancient Greek philosophers Plato and Aristotle. Experimental studies within this area began with the pioneering work of Ebbinghaus (1885). He employed nonsense syllables to study different forms of memory. Among his several discoveries was the fact that he could correctly recall seven syllables immediately after a single reading. At around the same time, James (1890) coined the term "primary memory" to designate this ability to maintain a small amount of information in consciousness for a brief period of time. In the middle of the 20th century, Miller (1956) also employed the expression "immediate memory" to refer to this same temporary memory system and confirmed that its temporary capacity consisted of seven plus or minus two units, whether the units are numbers, letters, words, or any other chunks of information.

A landmark in the study of memory was the Atkinson and Shriffrin (1968) model, one that included sensory memory, short-term memory (STM), and long-term memory (LTM). According to this model, information from the external word is first registered as sensory memory for a very brief period of time (200-500 ms) after the original stimulus ceases. Sensory memory of visual, auditory, and touch stimuli has also been termed iconic, echoic, and haptic memory, respectively. Because sensory memory is a short-lived phenomenon, this type of memory is not under cognitive control.

From sensory memory, a limited amount of the incoming information that was attended passes to STM, a capacity-limited, unitary memory store which temporarily keeps information for further processing. This STM corresponds to the "primary memory" or "immediate memory" defined by Ebbinghaus (1885) and James (1890), respectively. Information in STM decays after a few seconds if not rehearsed. However, if rehearsal occurs, then the information is consolidated into LTM, having an unlimited capacity to retain information for a long period of time.

The relationship between STM and LTM is mediated by two different mechanisms: consolidation and retrieval. Consolidation comprises the processes of transferring the information from a transitory STM system to a more permanent LTM system. It begins at the time of the learning experience and leads to a certain stabilization of the information. When consolidation occurs and the information has entered LTM, it fades from awareness. However, we know that the information has been successfully stored in LTM because we can bring it back to our consciousness through a retrieval mechanism. Therefore, retrieval comprises the search for specific information stored in LTM and the act of bringing it back to STM. Atkinson and Shiffrin (1971) equated STM to the content of consciousness: "In our thinking we tend to equate the short-term store with 'consciousness,' that is, the thoughts and information of which we are currently aware can be considered part of the contents of the short-term store" (p. 83).

However, results from studies of healthy subjects and neuropsychological studies of brain-damaged patients began to pose serious challenges to the serial model of memory proposed by Atkinson and Shriffrin (1968). These results challenged the idea that STM is a single storage system that needs to be activated

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as a whole to consolidate new information into LTM. To deal with these problems, different concepts of the STM model, envisioning different storage mechanisms within this system, began to emerge under the designation of working memory.

The term "working memory" was coined by Miller, Galanter & Pribram (1960) and used by Atkinson and Shriffrin (1968) as synonymous with STM. However, more recent literature has made a clear distinction between STM and working memory. Today, we define working memory as the ability to simultaneously process and keep information at a conscious level over a relatively short period of time to perform a cognitive task. The model was initially proposed by Baddeley and Hitch (1974) and remains the most influential framework within this area, despite the fact that several other working memory models have been proposed (e.g., Cowan, 2005; Engle, Kane & Tuholski, 1999; Ericsson & Kintsch, 1995). Although STM and working memory share a close relationship, both referring to transient memory, the concept of working memory replaced the concept of a unitary short-term storage system proposed by Atkinson and Shiffrin (1968) with the notion of a multiple-component structure. Moreover, the working memory model represents an interface between processing, storing, and manipulating information, as opposed to STM, which only has storage capacity.

The traditional working memory model postulates the existence of interacting components responsible for holding and manipulating verbal and visual-spatial information for a brief period of time. The phonological loop is responsible for the temporary retention of verbal material. Information is maintained in the phonological loop through a subvocal rehearsal process. Verbal information that is temporally stored in the phonological loop can be presented auditorially or visually, as long as the visual stimuli is nameable, such as letters, words, or numbers. An example of phonological loop function is the problem that we usually face when we have to memorize a phone number before we make a phone call. A few minutes after doing that, we can no longer remember the numbers. Because most of the first studies that were conducted with this temporary storage system employed verbal information, equating the phonological loop with STM and consequently the historical concepts of "primary memory" or "immediate memory," in which a limited amount of verbal information is held temporarily in consciousness, is possible.

The visual-spatial sketchpad represents another component of working memory and is responsible for the temporary retention of visual-spatial information. A good example of this working memory component is what happens when driving on a foggy road. We are constantly aware of the location of the cars, but as we continue to drive, we are unable to recall where each car was on the road. Another example of how the visual-spatial information employed by working memory is handled by the visual-spatial sketchpad is a popular concentration or memory game. The purpose of this game is to find the location of pairs of pictures that are face down after they had been reversed, memorized, and put back, facing down again. When a pair of figures is matched, remembering where these figures were located becomes difficult.

More recently, Baddeley (2000) proposed a new working memory component, termed the episodic buffer. This component is responsible for combining phonological, visual, and spatial information and integrating them with previously consolidated information in LTM into a unitary episodic representation. The episodic buffer corresponds to some sort of temporary episodic memory that can chunk information together according to our prior knowledge and thus enhance the capacity of STM, chunking information in units that make sense to the subject (Miller, 1956).

The phonological loop, visual-spatial sketchpad, and episodic buffer are connected to the central executive component of working memory. This fourth component acts as an attentional controller, coordinating the work of these three components that are considered slave systems to indicate the fact that their storage activity is directed by the central executive (Baddeley, 1996). This processing function includes directing attention, maintaining task goals, making decisions, and retrieving memories from LTM. Baddeley (1986, 1990) equated the central executive to the supervisory attentional system described by Norman and Shallice (1986). This system is responsible for allocating attention to task- or goal-relevant information while inhibiting task-irrelevant information.

Therefore, Baddeley's working memory model has three STM components—two of them are related to modality-specific information (i.e., verbal or visual-spatial) and the third is associated with multimodal information (i.e., the episodic buffer)—and a single processing component related to attention and executive function. The interaction between these four working memory components is necessary to perform

higher-level cognitive activities, such as mathematical thinking. Since Hitch's (1978) early article on multistep arithmetic problem solving, several reports on the critical role of working memory in mathematical problem solving have been published. Some of these evidences are discussed next.

WORKING MEMORY COMPONENTS THAT UNDERLIE MATHEMATICAL THINKING

Mathematical problem solving is a multidimentional task that most likely requires the activation of different working memory components. An initial inquiry in the study of the relationship between each working memory component and mathematical thinking was whether verbal or visual-spatial activation is necessary during mathematical thinking. The answer to this question appears to be complex. Although it is impossible to addresses this issue in the present work, we can illustrate the convolution of this topic with the report of well-know mathematician. Henri Poincaré (1948), for example, acknowledged the importance of language in his mathematical thinking, enforcing the idea that mathematical representation is a consequence of human linguistic competence. However, other distinguished mathematicians, such as Albert Einstein, emphasized non-verbal, language-independent processes related to analogical mental transformations and visual-spatial processing in his mathematical thinking. He stated that "words and language, whether written or spoken, do not seem to play any part in my thought processes. The psychological entities that serve as building blocks for my thought are certain signs or images, more or less clear, that I can reproduce and recombine at will" (Dehaene, Spelke, Pinel, Stanescu & Tsivkin, 1999).

Current empirical research that investigates the participation of each working memory component in mathematical thinking employs different methodologies. One strategy is to examine the correlations among several measurements of working memory and mathematical tasks. Based on individual differences across these variables, determining the extent to which each working memory component can predict different types of mathematical performance is possible. Another methodological design that can be employed to experimentally investigate this

problem is the dual-task paradigm. According to this procedure, a subject is required to perform two tasks simultaneously, called a primary or criterion task (e.g., solving simple mathematical problems) and a secondary task (e.g., any task that activates a component of the working memory system). The subject is also required to perform the primary or criterion task alone (single-task condition). A comparison of the dual-task condition with the single-task condition allows an evaluation of whether the same cognitive resources are shared between the primary (i.e., criterion) and secondary tasks. In other words, if the working memory component is required to solve a mathematical problem, then the secondary task should interfere with the primary mathematical task and thus reduce performance in the dual-task condition compared with the single-task condition. However, if the working memory component is not required to solve the mathematical problem, then no interference between the secondary and primary tasks should occur, and no difference between the dual- and single-task conditions should be observed.

An important issue in the study of mathematical thinking is that these skills depend on formal instruction, usually beginning at school entry. Therefore, most studies that investigate the relationship between working memory and mathematical ability focus on children of preschool or school age. These studies can employ children with normal learning development, mathematically precocious children, or even children who suffer from mathematical learning disabilities, a condition known as dyscalculia. Finally, developmental studies generally employ a cross-sectional design, in which different children of different ages are assessed at the same time. Longitudinal designs can also be employed and consist of studying the same group of children over a particular period of time.

What these studies clearly indicate is that working memory is strongly implicated in mathematical ability. This relationship depends on several variables, such as age, the difficulty of the mathematical problems, the type of instruction, and the way in which the problem is presented (Raghubar, Barnes & Hecht, 2010). Unfortunately, not completely clear is how different mathematical skills are handled by the four working memory components. Curiously, the contribution of the episodic buffer to mathematical achievement has not been investigated. The lack of studies devoted to the investigation of this particular relationship might be attributable to the

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fact that the concept of the episodic buffer is relatively new, and only a few measurements have been developed to evaluate the role of this working memory component in mathematical performance (Henry, 2010).

Numerous studies have investigated the participation of the phonological loop and visual-spatial scratchpad in mathematical performance among children with mathematical learning problems. The results indicate that both working memory and slave working memory components are involved in this developmental disorder. For example, Swanson & Jerman (2006) reported a meta-analysis of 28 studies that compared several cognitive features of children with and without mathematical disabilities. They found that differences between these two groups were related to verbal working memory after controlling for other variables, such as age and intelligence. These results might indicate that poor performance among children with mathematical learning problems is attributable to difficulties decomposing and understanding the mathematical problem (Henry & Maclean, 2003).

The visual and spatial representation of numerical information in working memory is also an important factor associated with mathematical learning disability. A few studies indicated that problems in spatial but not visual tasks might be associated with mathematical learning difficulties (Cornoldi, Venneri, Marconato, Molin & Montinari, 2003). Mammarella, Lucangeli & Cornoldi (2010) found that children with mathematical difficulties had lower scores on spatial tasks but not visual working memory tasks compared with a control group matched for verbal ability, age, gender, and sociocultural level. The same pattern of results was also reported recently by Passolunghi & Mammarella (2011) when children with mathematical learning disabilities were compared with another group of children with poor problem-solving skills.

As expected, deficits in the central executive component of working memory also appears to be an important factor responsible for mathematical problems because mathematical tasks place a heavy demand on this executive system. A lack of coordination of the many processes involved in counting or solving arithmetic problems might be attributable to a poor central executive component of working memory (Swanson & Olga, 2006). This appears to be consistent among children with mathematical learning deficiencies and children with persistently low achievement in

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mathematics. They appear to present problems understanding and representing numerical magnitude and difficulties retrieving basic arithmetic facts from LTM. These difficulties, in turn, cause a delay in learning mathematical procedures (Geary, 2011a).

Studies of healthy subjects also indicated the importance of all three of these working memory components in mathematical skills. The roles of the phonological loop and visual-spatial sketchpad appear to vary according to the complexity and content of the mathematical task. Accordingly, the phonological loop appears to be important for processes that involve the articulation of numbers, such as counting (Krajewski & Schneider, 2009) and verbal coding strategies during written arithmetic problem solving (Andersson, 2008). The visual-spatial sketchpad, in turn, appears to be associated with intuitive aspects of number processing and calculation (de Hevia, Vallar & Girelli, 2008) and a broader number of mathematical domains (De Smedt, Janssen, Bouwens, Verschaffell, Boets & Ghesquière, 2009).

The role of the phonological loop and visual-spatial sketchpad in healthy children with no mathematical problem-solving difficulties might change with age and mathematical problem experience (Meyer, Salimpoor, Wu, Geary & Menon, 2010). For example, McKenzie, Bull & Gray (2003) used the dual-task procedure to investigate phonological and visual-spatial working memory components in a simple arithmetic task in younger (6-7 years old) and older (8-9 years old) children. They found that younger children were not affected by phonological interference, whereas visual-spatial interference had a significant effect on mathematical performance. The results indicated that the older children were affected by both phonological and visual-spatial interference. Therefore, younger children appeared to employ only the visual-spatial sketchpad, whereas older children relied on both the visual-spatial sketchpad and phonological loop components of working memory.

The role of the visual-spatial sketchpad in mathematical thinking appears to play an increasingly important role in mathematical ability during later stages of development, such as at 9 to 10 years of age (Holmes, Adams, & Hamilton, 2007), 11 and 12 years of age (Henry & Maclean, 2003), 12 to 13 years of age (Dark & Benbow, 1990), and 15 to 16 years of age (Reuhkala, 2001). The phonological loop,

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in turn, appears to be an important element in mathematical performance when the task has been well-learned (Raghubar et al., 2010).

The central executive also plays a pivotal role in mathematical thinking in healthy children. In a five-year longitudinal study, Geary (2011b) reported that the central executive component of working memory predicted mathematical performance. A longitudinal study of children in the first and second grade also found that the central executive was the unique predictor of mathematics problem-solving achievement (De Smedt et al., 2009).

Finally, a study of subjects who excelled in mathematics performance supported the primary conclusion that successful mathematical problem solving requires the activation and optimal use of working memory resources. Precocious first-grade children with extremely good performance on mathematical problems had higher working memory ability in all three components (Hoard, Geary, Byrd-Craven, & Nugent, 2008) or at least in the central executive system (Swanson, 2006) compared with typical children of the same age. Evidence has also shown that adolescents with exceptional mathematical abilities had greater activity in the visual-spatial and central executive components of working memory (Desco et al., 2011).

The interplay between these different working memory components during number processing has been proposed by the triple-code theory posited by Dehaene and colleagues (Dehaene, 1992; Dehaene, Bossini, & Giraux, 1993; Dehaene & Cohen, 1995). According to this model, numerical information can be mentally represented and manipulated in three different forms. First, the verbal representation of numbers appears as strings of words (e.g., thirty-seven) in the phonological loop. Second, a visual Arabic representation of the number occurs as a string of numerals (e.g., 37) in the visual-spatial scratchpad. Finally, an analogical spatial representation of the number expresses the number's magnitude in a mental number line so that its magnitude can be compared with other numbers (e.g., knowledge that 37 is bigger than 7 and smaller than 50 and is approximately one-third of the way between 0 and 100). The hypothesis that working memory capacity is directly associated with mathematical performance suggests an overlap between the neural circuitries involved in working memory and mathematical thinking. Below we present evidences that support this possibility.

NEURAL CIRCUITRIES INVOLVED IN WORKING MEMORY AND MATHEMATICAL THINKING

Several clinical and neuroimaging studies that have used brain-damaged patients and healthy subjects have successfully investigated the neural circuitry that underlies working memory and mathematical thinking. Working memory is a more general system, the functioning of which involves a sensory-perceptual mechanism that is necessary to process different stimulus modalities that come from the external world. Auditory and visual stimuli are the two sensory components of working memory. Auditory stimuli related to verbal information are processed by the primary auditory cortex in the temporal lobe, whereas visual-spatial information is processed by the primary visual cortex located in the occipital lobe. Visual and auditory information are integrated at higher level through efferent projections from these primary sensorial areas to the posterior parietal cortex. Hemispheric specialization exists, according to the modality dimension of the information processed by the working memory system. The left parietal cortex is more active during verbal tasks, whereas the right parietal cortex is preferentially activated during nonverbal spatial tasks (Wager & Smith, 2003).

The posterior parietal cortex can be divided into the superior parietal lobule and inferior parietal lobule, including the angular and supramarginal gyri, separated by the horizontal intraparietal sulcus. The inferior parietal lobule appears to be associated with auditory encoding, especially speech processing, such as in phonological discrimination and in identification task. The superior parietal lobule, in turn, appears to be related to the processing of visual-spatial information. For example, lesions within this area might lead to visuospatial hemineglect (Driver & Mattingley, 1998). Reciprocal projections between the superior and inferior parietal lobules are important for multisensory integration within the parietal lobe that might translate visually processed information into verbal or phonological code, such as in reading tasks.

These neural networks within the posterior parietal cortex might be also related to the episodic buffer, responsible for binding unimodal information into single multimodal units. The episodic buffer is also responsible for the retrieval of

information from LTM according to its multidimensional representation. Consistent with this function, the hippocampus (Berlingeri et al., 2008; Rudner, Fransson, Ingvar, Nyberg & Rönnberg, 2007) in conjunction with the prefrontal cortex might also underlie the neural circuitry of the episodic buffer.

The prefrontal cortex is probably the most important neural substrate within this circuitry responsible for the central executive component of working memory (D'Esposito, Detre, Alsop, Shin, Atlas, & Grossman, 1995). The prefrontal cortex is divided into medial and lateral surfaces, the latter consisting of ventrolateral, dorsolateral, and anterior prefrontal regions. The dorsolateral and ventrolateral prefrontal regions have distinct neuronal specializations and are associated with different domains of working memory. The dorsolateral prefrontal cortex appears to be responsible for holding information in consciousness when it is no longer available in the environment but still necessary to perform a certain cognitive task, such as directing and changing attention to the internal representation of sensory stimuli. This ability of working memory to manipulate information is likely accomplished by bidirectional projections that the dorsolateral prefrontal cortex maintains with the posterior parietal cortex (Crone, Wendelken, Donohue, van Leijenhorst, & Bunge, 2006).

The dorsolateral prefrontal cortex also plays a role in rehearsal mechanisms through projections that it maintains with the ventrolateral prefrontal cortex. This area lies within the left hemisphere and corresponds to Broca's area. It is highly activated in working memory tasks that require subvocal rehearsal and other verbal tasks. The ventrolateral prefrontal cortex within the right hemisphere, in contrast, is more active during visual-spatial tasks (D'Esposito et al., 1995). Therefore, the ventrolateral prefrontal cortex might have lateralized specialized functions that might be responsible for different working memory components. The phonological loop appears to be associated with the functioning of the left hemisphere, whereas the visual-spatial sketchpad is associated with the functioning of the right hemisphere (Smith & Jonides, 1999).

Brain lesion and neuroimaging data suggest considerable overlap between the neural circuitry involved in working memory and mathematical skills. The ability to solve mathematical problems has typically been associated with cortical areas within

the left hemisphere associated with linguistic ability. That is the case with Broca's area (i.e., ventrolateral prefrontal area of the left hemisphere) and its involvement in both linguistic and arithmetic function (Delazer et al., 2005). Accordingly, patients with motor aphasia might also present calculation deficits (Dehaene & Cohen, 1997).

However, other nonverbal brain structures within the left and right hemispheres have also been associated with mathematical thinking. The role of the posterior parietal cortex in mathematical ability has been inferred from studies of patients with brain lesions. For example, Gerstmann's Syndrome (Gerstmann, 1940) involves damage to the left inferior parietal lobule and is associated with symptoms related to dyscalculia and finger agnosia (i.e., a disability in the mental representation of fingers), thus imposing difficulty using the fingers to count. Patients with lesions of the left or right posterior parietal cortex also presented different forms of dyscalculia (Cipolotti, Butterworth, & Denes, 1991; Molko et al., 2003). Furthermore, the temporary disruption of neural activity following transcranial magnetic stimulation either in the left or right parietal cortex disrupted performance in several mathematical problems (Cappelletti, Barth, Fregni, Spelke & Pascual-Leone, 2007). Finally, several neuroimaging studies have shown that the posterior parietal cortex, including both the superior and inferior parietal lobules, is implicated in mental calculation (Dehaene et al., 1999).

Because the posterior parietal cortex is also involved in working memory, these findings suggest neural overlap between working memory and mathematical ability. Consistent evidence also suggests the existence of a brain region exclusively specialized for number processing. Two meta-analyses (Dehaene, Piazza, Pinel, & Cohen, 2003; Cohen-Kadosh, Lammertyn & Izard, 2008) found that the horizontal intraparietal sulcus in both hemispheres represents a specialized brain area for number processing. This region is systematically activated in all number tasks, independent of the modality of the information, regardless of whether the numerals were written (visual), spoken (auditory), or even spelled out. The involvement of the horizontal intraparietal sulcus in the processing of numerical information is so striking that abnormalities in this region might represent a biological marker of developmental dyscalculia (Rubinsten & Henik 2009; Butterworth, 2010).

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Interestingly, activity in the horizontal intraparietal sulcus is also associated with the working memory system. For example, individual differences in horizontal intraparietal sulcus activity is associated with working memory capacity in adults (Todd & Marois 2005) and comparisons between children and adults (Klingberg, Forssberg, & Westerberg, 2002). Recently, Dumontheil and Klingberg (2011) found that greater activation in the left but not right horizontal intraparietal sulcus during a visual-spatial working memory task was associated with poorer arithmetical performance 2 years later. Therefore, working memory and mathematical ability might share important neural circuitries within the posterior parietal cortex.

Consistent with this possibility, distinct neural circuitries within the posterior parietal cortex might be associated with different working memory components involved in mathematical performance (Dehaene et al., 2003). Projections from the neural regions within the horizontal intraparietal sulcus to the inferior parietal lobule of the left hemisphere form a network with Broca's area in the prefrontal cortex and might be related to number manipulation in verbal form, thus representing a possible interaction between the phonological loop and central executive of working memory. Bilateral projections from the area within the horizontal intraparietal sulcus to the superior parietal lobule that in turn sends connections to the occipital cortex might be involved in the visual-spatial and attentional aspects of mental calculation. Activity within the primary and secondary visual occipital cortices suggests the importance of visual representational mechanisms in mathematical thinking.

Dehaene et al. (1999) also found that the left and right posterior parietal cortex might play differential roles in mathematical ability. The left inferior parietal lobule is involved in exact arithmetic calculation and is language-dependent. Approximate arithmetic, in contrast, is not language-dependent and relies primarily on a quantity representation implemented in the visual-spatial networks of the left and right parietal cortex. Differences in mathematical reasoning between the left and right posterior parietal lobe have also been reported by Cappelletti, Lee, Freeman, & Price (2010). They found that the right posterior parietal cortex was involved in conceptual decisions in selective tasks that involve numbers, whereas left posterior parietal activation did not depend on whether the information was extracted and compared with numbers or object names.

The role of the right hemisphere in mathematical skills has also been observed in mathematically gifted students. Subjects with a high level performance in mathematical problem solving presented higher activation of the right prefrontal and medial temporal areas compared with a group of non-mathematically gifted subjects (Pesenti et al., 2001). However, the main finding from this study was the bilateral neural activity that mathematically gifted subjects exhibited during mathematical thinking. Higher connectivity between the left and right hemispheres in fronto-parietal regions appears to be the main characteristic of subjects with high mathematical abilities (Alexander et al., 1996; Singh & O'Boyle, 2004; O'Boyle et al., 2005).

Notably, fronto-parietal networks related to working memory and mathematical thinking can interact with other brain circuitries involved in different emotional and motivational systems. This interaction can enhance or reduce working memory capacity and thus mathematical problem-solving performance. For example, mathematically gifted subjects presented high levels of motivation and consequently a high amount of training in mathematical problems that in turn might enhance working memory resources (Dehaene, 2001). Interactions with other neural circuitries involved in anxiety, in contrast, might negatively impact these fronto-parietal networks and thus reduce working memory resources and mathematical performance. The negative impact of anxiety on cognitive tasks related to mathematical problem solving is discussed next.

ANXIETY, WORKING MEMORY, AND MATHEMATICAL THINKING

Imagine that you have to solve the multiplication problem presented at the beginning of this paper under an extremely dangerous condition, such as during a car accident in which you are involved. Your performance would certainly be well below your mathematical ability because of the high anxiety triggered by this dangerous situation. This example illustrates how high levels of anxiety can interfere with our working memory capacity and thus lower our performance on mathematical tasks.

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Anxiety can be defined as an emotional or motivational state that can be unpleasant or aversive and is triggered by a real or potentially threatening environmental situation. Anxiety produces a series of physiological and behavioral responses that might help deal with the threatening situation. The relationship between anxiety and task performance has been suggested to follow an inverted U-shaped curve (Yerkes & Dodson, 1908). This relationship, known as Yerkes-Dodson Law, holds that every task has an optimal point of anxiety for its best performance. Levels below or above this point tend to impair performance on the task. The Yerkes-Dodson Law also suggests that the optimal anxiety level for the best task performance depends on the degree of task difficulty. Thus, a harder task is associated with a lower optimal level of anxiety. Conversely, an easier task is associated with a higher optimal level of anxiety. However, excessively high levels of anxiety always impair task performance, independent of how easy the task is.

Cognitive tasks have a certain degree of difficulty so that small levels of anxiety can disrupt cognitive task performance. Indeed, a negative relationship appears to exist between anxiety and mathematical performance (Eysenck, Derakshan, Santos & Calvo, 2007). Interference with working memory function seems to be the main reason for the adverse effects of anxiety on mathematical problem solving (Ashcraft & Knause, 2007). Individuals with elevated trait anxiety experience deficits in attention shifting and the inhibition of irrelevant information, leading to poor performance on mathematical problems (Eysenck et al., 2007). According to this view, anxiety consumes a portion of the limited resources of working memory necessary to solve mathematical problems. This situation resembles a dual-task paradigm, in which anxiety functions as a resource-demanding secondary task that negatively impacts the primary or criterion mathematical task (Ashcraft & Krause, 2007).

The desire to perform as well as possible under stressful or high-pressure circumstances also reduces mathematical performance because of working memory impairment. Interestingly, subjects with higher working memory capacity under normal conditions performed worse than subjects with lower working memory capacity under a high-pressure test situation (Beilock, 2008). These results suggest that individuals with high and low working memory capacities employ different

strategies to solve complex mathematical problems. Individuals with high working memory capacity rely on more demanding procedures to solve mathematical problems. High-pressure situations impact working memory resources, making them susceptible to failure. In contrast, individuals with low working memory capacity rely on shortcuts to solve these same mathematical problems because of their working memory limitations. Therefore, pressure-induced working memory resource consumption does not disrupt their mathematical performance (Beilock, 2008).

Although there are still several debates, it has been suggested that mathematical activities *per se* are well known to cause anxiety (Ashcraft, 1995; Hembree, 1990; Richardson & Suinn, 1972). This effect seems to be specific to mathematical tasks and occurs independently of any high-pressure circumstance or task difficulty. Mathematics anxiety is characterized by feelings of apprehension and tension concerning the manipulation of numbers and completion of mathematical problems in various contexts (Richardson & Suinn, 1972). People who suffer from mathematical anxiety have problems counting and interpreting mathematical problems but exhibit no deficits in other difficulty-matched non-mathematical tasks (Faust et al., 1996). Poor mathematical performance in subjects who suffer from mathematical anxiety is associated with limitations in the central executive component of working memory (Andersson, 2008).

Subjects with mathematical anxiety typically present anxiety reactions before performing the task. In a recent neuroimaging study, Lyons and Beilock (2011) found that not all mathematical anxiety subjects performed equally poorly on a mathematical task. Moreover, mathematical deficits in subjects who suffer from mathematical anxiety could be predicted by bilateral activation of fronto-parietal circuitries that involve the dorsolateral prefrontal cortex before the beginning of the mathematical task. The dorsolateral prefrontal cortex is also associated with the interpretation of potentially threatening situations (Bishop, 2007). Therefore, greater activation of the neural circuitries involved in working memory and mathematical thinking in the dorsolateral prefrontal cortex before beginning a mathematical task may be associated with a better ability of mathematical anxiety subjects to inhibit their negative reactions to the upcoming mathematical task and thus overcome this mathematics-specific deficit.

CONCLUSIONS

Mathematical thinking is an important mental function for everyday life. It requires highly diverse cognitive abilities, ranging from decomposing and understanding the mathematical problem verbally to abstract symbol manipulation in a visual-spatial representation. For example, learning to read can occur independently of any mathematical knowledge. Nonetheless, reading is a necessary condition to solve mathematical problems. Therefore, different cognitive domains that store and manipulate verbal and visual-spatial information while solving a mathematical problem represent crucial factors for successful performance on these tasks.

A considerable amount of empirical evidence from the past 30 years clearly indicates that different working memory components are critically involved in the solution of mathematical problems that require more than just memory retrieval. Indeed, working memory is increasingly more involved in mathematical reasoning as more intermediate steps are required to solve a mathematical problem (Beilock, 2008). Neuroimaging studies corroborate this premise. Although the precise neural circuitry that underlies mathematical thinking is not completely clear, several studies have reported that working memory and mathematical tasks likely recruit the same brain structures within the posterior parietal and prefrontal cortices.

Understanding the neuropsychological mechanism and neural circuitries of mathematical thinking might help develop measures that can improve mathematical problem-solving achievement. The fact that adequate mathematical performance requires an optimal working memory capacity suggests that improving working memory resources might enhance mathematical performance. For example, Witt (2011) recently reported that children who received working memory training exhibited a significant improvement in mathematical problem solving compared with matched subjected who did not receive working memory training.

Emotional and motivational aspects are also important factors in mathematical thinking. For example, mathematically gifted students experience feelings of reward when solving mathematical problems, causing continuous training in these mathematical tasks. However, other people perceive mathematical tasks as

threatening and an anxiogenic condition. Anxiety reactions to this type of problem can be so high that they might represent a major threat to mathematical performance, probably because of the fact that anxiety consumes working memory resources. The negative impact of anxiety on mathematical achievement may arise even before the subject begins to solve the mathematical problem. Therefore, controlling the negative feelings that precede mathematical performance might be the best way to deal with this type of mathematical performance failure. Accordingly, appropriate mathematical thinking certainly involves the good management of one's working memory resources and adequately dealing with anxiety reactions that might occur before solving the mathematical task.

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